

# Long Time Accumulation Algorithm Based on KT-Radon and CPF

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**Abstract**—In order to solve the problem of range migration and Doppler spread in the detection of target with higher order motion during long-time integration, in this paper, an improved signal accumulation method based on Keystone transform(KT), Radon Transform(RT) and Cubic phase function(CPF) is proposed. Firstly, the algorithm uses KT-Radon to correct the range migration of the echo, then by using CPF to estimate high order phase, the acceleration and the jerk of the target is compensated, at last the algorithm coherently integrates the target's energy by Fourier transform (FT). Compared with the GKTGDP[12] algorithm, the proposed method only needs one-dimensional search for acceleration and jerk. Compared with the KT-CPF [14]algorithm, there is no limitation on search range of Doppler fold factor in this proposed method.

**Keywords**—high order; long-time integration; range migration; Doppler spread.

## I. INTRODUCTION

In the complex engagement environment, the modern radar is needed to detect the high speed, high maneuverability and stealthy target [1-4]. However since the RCS of the stealthy target significantly decreases, the target echo will be too weak to detection. The long-time coherent integration can improve SNR of the target echo [5-7]. Unfortunately, during the long-time integration, the echo of the high-speed maneuvering target will have range migration(RM) and Doppler spread(DS) induced by target's complex motions, which will seriously affect the accumulation of target energy.

In order to solve the RM in the synthetic aperture radar (SAR), the first-order Keystone Transform(KT) is proposed to correct RM of the target [8]. Then Zhang and Zeng applied the KT to the weak target detection of pulse Doppler (PD) radar[9]. Xu et al [5] proposed a long-time integration method for radar target echo based on Radon-Fourier transform (RFT). For the target with radial constant acceleration motion, the second order KT can be used to correct the RM caused by the radial acceleration of the target[10-11]. However, all of the above methods are not suitable for high speed targets with high order motion. Long-time integration method for high speed maneuvering target based on generalized RFT transform (GRFT) is proposed in [7]. This method can effectively accumulate energy of the high speed target with jerk motion. But the

algorithm needs four dimensional search ,therefore the computation load is very heavy. Reference [12] proposes a long time accumulation algorithm based on GKTGDP which could effectively accumulate the energy of the target with jerk motion and reduce the computation load, but it needs to jointly search the acceleration and the jerk of the target.

Focusing on the maneuvering target with jerk motion, a new method of signal accumulation based on KT is proposed in this paper, which uses KT and RT to correct the RM of the echo, then estimating the acceleration and the jerk based on the CPF to compensating high order phase, at last Fourier transform (FT)is carried out to achieve the coherent integration. Compared with the existing algorithms, the presented method can acquire same coherent integration performance and only needs one-dimensional search for acceleration and jerk. There is no limitation on search range of Doppler fold factor too.

This remainder of this paper is organized as follows. In Section II, we analyses the signal model of a target with high order motion. In Section III, a new algorithm for long-time integration of signals based on KT,RT and CPF is proposed. In Section IV, we analyses the computational complexity of the algorithm, and compares the results with the three order RFT algorithm and the GKTGDP algorithm. In Section V, simulation experiments are performed to verify the algorithm. At last, a conclusion is given in Section V.

## II. ECHO SIGNAL

Suppose that the radar signal envelope is a linear frequency modulated(LFM) signal, i.e.:

$$s(\tau) = \text{rect}\left(\frac{\tau}{T_p}\right) \exp(j\pi\gamma\tau^2) \quad (1)$$

Where  $\text{rect}(u) = \begin{cases} 1, & |u| \leq \frac{1}{2} \\ 0, & |u| > \frac{1}{2} \end{cases}$ ,  $\tau$  is the fast time,  $\gamma$  is chirp rate

and  $T_p$  is the pulse width.

Ignoring the motions above third-order, the instantaneous radial distance between the radar and the target with jerk motion is:

$$R(t_m) = R_0 + vt_m + a_1 t_m^2 + a_2 t_m^3 \quad (2)$$

The radial distance of initial target is  $R_0$ ,  $v$  represent the target speed,  $a_1$  is the radial acceleration and  $a_2$  is the radial jerk.  $t_m = m \cdot T_r$  denotes the slow time,  $m = -\frac{N-1}{2}, \dots, -1, 0, 1, \dots, \frac{N-1}{2}$ , where  $m$  is the number of pulse, and  $T_r$  is Radar pulse repetition period,  $N$  is the total number of radar pulses.

For high speed targets, we set  $|v|/c \ll 1$  ( $c$  is the speed of light), The echo signal received by the radar is [11]:

$$s_r(\tau, t_m) = Q_0 \text{rect}\left(\frac{\tau - 2R(t_m)/c}{T_p}\right) \exp\left[j\pi\gamma\left(\tau - \frac{2R(t_m)}{c}\right)^2\right] \times \exp\left[-j\frac{4\pi R(t_m)}{\lambda}\right] \exp\left(-j\frac{4\pi v\tau}{\lambda}\right) \quad (3)$$

Where  $Q_0$  is the reflection coefficient,  $\lambda = c/f_c$  denote the wave length,  $f_c$  is the Radar carrier frequency.

The received signal after pulse compression, in the range frequency-slow time ( $f - t_m$ ) domain can be expressed as:

$$S_{rm}(f, t_m) = Q_0 \text{rect}\left(\frac{f + f_d/2}{B - f_d}\right) \exp(-j\frac{\pi}{\gamma} f_d^2) \times \exp(-j\frac{2\pi f_d f}{r}) \exp[-j4\pi \frac{f + f_c + f_d}{c} R(t_m)] \quad (4)$$

Where  $f_d = 2v/\lambda$ .

Substituting (2) into (4), yields:

$$S_{rm}(f, t_\beta) = Q_1 \exp[-j2\pi(\frac{f_d}{\gamma} + \frac{2R_0}{c})f] \times \exp[-j4\pi \frac{f + f_c + f_d}{c} v t_m] \exp[-j4\pi \frac{f + f_c + f_d}{c} a_1 t_m^2] \times \exp[-j4\pi \frac{f + f_c + f_d}{c} a_2 t_m^3] \quad (5)$$

Where  $Q_1 = Q_0 \text{rect}\left(\frac{f + f_d/2}{B - f_d}\right) \exp(-j\frac{\pi}{\gamma} f_d^2) \times \exp(-j\frac{4\pi(f_c + f_d)R_0}{c})$ .

When the pulse repetition frequency (PRF) of the radar system is low and the moving velocity of the target is high, undersampling would occur. therefore the real velocity of the target can be expressed as:

$$v = v_0 + K v_{amb} \quad (6)$$

Where  $v_{amb}$  is blind speed,  $v_{amb} = PRF \lambda / 2$ .  $v_0 = \text{mod}(v, v_{amb})$  is the folded Doppler frequency. And  $|v_0| < v_{amb}/2$ ,  $K$  is fold factor.

Substituting (6) into (5), yields:

$$S_{rm}(f, t_\beta) = Q_1 \exp[-j2\pi(\frac{f_d}{\gamma} + \frac{2R_0}{c})f] \times \exp[-j4\pi \frac{f + f_c + f_d}{c} v_0 t_m] \exp[-j4\pi \frac{f + f_c + f_d}{c} K v_{amb} t_m] \times \exp[-j4\pi \frac{f + f_c + f_d}{c} a_1 t_m^2] \exp[-j4\pi \frac{f + f_c + f_d}{c} a_2 t_m^3] \quad (7)$$

Set  $\xi = \frac{(f + f_d)}{f_c}$ , because the  $f_d \ll f_c$ , it can ignore the effect of  $f_d$  on  $\xi$ ,  $\xi$  is simplified for about  $\xi \approx \frac{f}{f_c}$ . so the formula (7) can be written as:

$$S_{rm}(f, t_\beta) = Q_1 \exp[-j2\pi(\frac{f_d}{\gamma} + \frac{2R_0}{c})f] \times \exp[-j\frac{4\pi v_0 t_m}{\lambda}(1 + \xi)] \exp[-j\frac{4\pi K v_{amb} t_m}{\lambda}(1 + \xi)] \times \exp[-j\frac{4\pi a_1 t_m^2}{\lambda}(1 + \xi)] \exp[-j\frac{4\pi a_2 t_m^3}{\lambda}(1 + \xi)] \quad (8)$$

Due to  $\exp[-j\frac{4\pi K v_{amb} t_m}{\lambda}] = 1$ , the formula (8) can be simplified as:

$$S_{rm}(f, t_\beta) = Q_1 \exp[-j2\pi(\frac{f_d}{\gamma} + \frac{2R_0}{c})f] \times \exp[-j\frac{4\pi v_0 t_m}{\lambda}(1 + \xi)] \exp[-j\frac{4\pi K v_{amb} t_m}{\lambda} \xi] \times \exp[-j\frac{4\pi a_1 t_m^2}{\lambda}(1 + \xi)] \exp[-j\frac{4\pi a_2 t_m^3}{\lambda}(1 + \xi)] \quad (9)$$

In equation (9), the first exponential term is a frequency offset, which is caused by the high speed motion and range of the target. The second exponential term is the Doppler frequency term related to the velocity of the target. The third exponential term is related to Doppler frequency fold factor. The fourth exponential term is the Doppler frequency modulation term caused by the acceleration of the target. The fifth exponential term is the quadratic Doppler frequency modulation term caused by the jerk.

In the first exponential term, the range frequency  $f$  isn't coupled with slow time  $t_m$ , so it does not affect the energy accumulation of the target. The coupling relationship between  $f$  and  $t_m$  exists in the latter four index terms, which will cause the RM. At the same time, the quadratic and cubic terms of slow time  $t_m$  in the fourth and fifth exponential terms will cause Doppler spread(DS). Both the RM and the DS will affect the accumulation of the target energy.

### III. LONG TIME ACCUMULATION ALGORITHM BASED ON KTR AND CPF

In this section, a method based on KT, RT and CPF is proposed to eliminate the RM and achieve the coherent integration for the maneuvering target with jerk motion

## A. Range migration correction

### 1) Keystone transform(KT)

The KT is used to correct the RM of the target echo, and its transformation formula is as follows:

$$t_m = \left(\frac{f_c}{f + f_c}\right)t_\beta \quad (10)$$

Substituting (10) into (9), yields:

$$\begin{aligned} S_{rm}(f, t_\beta) &= Q_1 \exp[-j2\pi\left(\frac{f_d}{\gamma} + \frac{2R_0}{c}\right)f] \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \\ &\times \exp[-j\frac{4\pi K v_{amb} t_\beta}{\lambda} \frac{f}{f + f_c}] \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda(1+\xi)}] \\ &\times \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda(1+\xi)^2}] \end{aligned} \quad (11)$$

For narrowband radar, because  $f \ll f_c$ , yield:

$$\frac{f}{f + f_c} \approx \frac{f}{f_c}, \quad \frac{f_c}{f + f_c} \approx 1 \quad (12)$$

Substituting (12) into (11), yields:

$$\begin{aligned} S_{rm}(f, t_\beta) &= Q_1 \exp[-j2\pi\left(\frac{f_d}{\gamma} + \frac{2R_0}{c}\right)f] \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \\ &\times \exp[-j\frac{4\pi K v_{amb} t_\beta}{\lambda} \frac{f}{f_c}] \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda}] \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda}] \end{aligned} \quad (13)$$

then converting it into fast time-slow time  $(\tau, t_m)$  domain by inverse Fourier transform(IFT), yields:

$$\begin{aligned} S_{rm}(\tau, t_\beta) &= Q_2 \sin c\left[B\left(\tau - \frac{f_d}{\gamma} - \frac{2R_0}{c} - \frac{2K v_{amb} t_\beta}{\lambda f_c}\right)\right] \\ &\times \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda}] \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda}] \end{aligned} \quad (14)$$

It can be seen from the formula (14), the peak position of target echo envelope will still change with slow time. This is due to the Doppler fold factor of target echo. Therefore, it is necessary to compensate the Doppler fold factor to completely remove the RM of target echo.

### 2) Radon transform(RT)

Let  $f(x, y)$  is a function of a two-dimensional plane, the line integral of  $f(x, y)$  along the line L can be expressed as:

$$R(\rho, \theta) = \iint f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy \quad (15)$$

Where  $\rho$  is the distance between the straight L and the origin of the coordinates,  $\theta$  is the angle between the straight L and the X axis.  $f(x, y)$  is the value of point  $(x, y)$ , and delta  $(x)$  is Dirac's function, which can be expressed as:

$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \quad (16)$$

The Radon transformation can be seen as the sum of the projection of the point intensity on the line L on the image. As can be seen from the formula (14), the difference between the peak position of the adjacent pulse in the fast time is:

$$\Delta peak \approx \frac{2K v_{amb} PTR}{\lambda f_c} = \frac{K}{f_c} \quad (17)$$

So the formula (14) in the  $(T, t_m)$  plane will form a line. It is known from the previous definition that the slope A of the diagonal line in the graph can be represented as:

$$A = \frac{K f_s}{f_c} \quad (18)$$

It is known from the nature of RT, When RT is done along the vertical line after KT, it will has maximum value in the RT domain. Therefore, Doppler fold factor can be estimated based on the maximum value of RT domain. The estimation formula is:

$$\tilde{K} = \tan \left\{ \arg \max_{\theta} \left| \text{Radon} \left( \text{abs} \left[ S_{rm}(\tau, t_\beta) \right], \theta \right) \right| \right\} f_c / f_s \quad (19)$$

Finding out the estimated value of the fold factor  $\tilde{K}$ , design phase compensation function  $\exp[j\frac{4\pi \tilde{K} v_{amb} t_\beta}{\lambda} \frac{f}{f_c}]$ , multiplying the compensation function by the formula (13), yields:

$$\begin{aligned} S_{rm}(f, t_\beta) &= Q_1 \exp[-j2\pi\left(\frac{f_d}{\gamma} + \frac{2R_0}{c}\right)f] \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \\ &\times \exp[-j\frac{4\pi(K - \tilde{K}) v_{amb} t_\beta}{\lambda} \frac{f}{f_c}] \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda}] \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda}] \end{aligned} \quad (20)$$

The estimated value of the fold factor is  $K = \tilde{K}$ . The formula (20) can be simplified to:

$$\begin{aligned} S_{rm}(f, t_\beta) &= Q_1 \exp[-j2\pi\left(\frac{f_d}{\gamma} + \frac{2R_0}{c}\right)f] \\ &\times \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda}] \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda}] \end{aligned} \quad (21)$$

then converting it by IFT, yields:

$$\begin{aligned} S_{rm}(\tau, t_\beta) &= Q_2 \sin c\left[B\left(\tau - \frac{f_d}{\gamma} - \frac{2R_0}{c}\right)\right] \exp[-j\frac{4\pi v_0 t_\beta}{\lambda}] \\ &\times \exp[-j\frac{4\pi a_1 t_\beta^2}{\lambda}] \exp[-j\frac{4\pi a_2 t_\beta^3}{\lambda}] \end{aligned} \quad (22)$$

The above methods combine KT and RT to solve the Doppler ambiguity problem caused by the high velocity of target in the process of Keystone transformation. The target echo is successfully corrected to the same range unit.

## B. Doppler frequency correction

In the previous sections, we remove the RM of the target echo through the KT-RT method. At this point, the echo envelope is located in the same range unit, but there is a Doppler frequency spread caused by the acceleration and the jerk. Next we estimate the acceleration and the jerk of the target by CPF (Cubic Phase Function) method, and construct the phase compensation function to remove the high-order phase term of the echo signal, so as to eliminate the influence of DS. After correcting the RM, The maximum energy of the distance unit of the target is maximum, we take the distance

unit of the target to estimate the target parameter. The value formula is:

$$S_{rm}(t_\beta) = \arg \max_{\tau} \left| \text{sum} \left( \text{abs} \left[ S_{rm}(t_\beta, \tau) \right] \right) \right| \quad (23)$$

For simplified expression, the signal of the selected range unit can be expressed as:

$$S_{rm}(t_\beta) = Q_3 \exp[-j \frac{4\pi v_0 t_\beta}{\lambda}] \exp[-j \frac{4\pi a_1 t_\beta^2}{\lambda}] \times \exp[-j \frac{4\pi a_2 t_\beta^3}{\lambda}] \quad (24)$$

Where  $Q_3 = Q_2 \sin c[B(\tau - \frac{f_d}{\gamma} - \frac{2R_0}{c})]$ ,  $Q_3$  is a constant.

The cubic phase function (CPF) of the  $S_{rm}(t_\beta)$  is designed as follows:

$$CPF(\beta, \Omega) = \sum_{k=0}^{N/2} S_{rm}(t_\beta + t_k) S_{rm}(t_\beta - t_k) \times S_{rm}^*(t_\beta) S_{rm}^*(t_\beta) \exp(j \frac{4\pi}{\lambda} \Omega k^2 T_k^2) \quad (25)$$

Where  $t_k = kT_\beta$ ,  $T_\beta$  is a pulse time interval after KT.  $\Omega$  is the Instantaneous Frequency Rate (IFR).

$\Omega$  is defined as  $\Omega = \frac{d^2 \varphi(n)}{dn^2}$ ,  $\varphi(n)$  is the phase of the

signal,  $\Omega$  is the two order derivation function of the phase

Substituting (24) into (25), yields:

$$CPF(\beta, \Omega) = Q_3^4 \sum_{k=0}^{(N-1)/2} \exp[-j \frac{4\pi k^2 T_k^2}{\lambda} (2a_1 - 6a_2 t_\beta - \Omega)] \quad (26)$$

From the formula (25), the peak position of CPF [13] is:

$$\Omega = 2a_1 + 6a_2 t_\beta = 2a_1 + 6a_2 \beta T_\beta \quad (27)$$

Therefore, the estimated values of  $a_1$  and  $a_2$  can be derived according to the peak value of CPF at two different time. The specific steps of the estimation are as follows:

First, the peak positions of two different moments,  $a_1$  and  $a_2$ , are calculated:

$$\hat{\Omega}_1 = \arg \max_{\Omega} |CPF(\beta_1, \Omega)| \quad (28)$$

$$\hat{\Omega}_2 = \arg \max_{\Omega} |CPF(\beta_2, \Omega)| \quad (29)$$

When the value of  $\beta_1$  is 0, it can be obtained:  $\Omega = 2a_1$ . So the estimated value of  $a_1$  is:

$$a_1 = \frac{\Omega}{2} \quad (30)$$

The value of  $\beta_2$ , after a lot of calculation, When  $\beta_2 \approx 0.11N$ , the mean square error of acceleration value is Minimum. At this point, the estimated value of  $a_2$  is:

$$\hat{a}_2 = (\hat{\Omega}_2 - \hat{\Omega}_1) / 6\beta_2 T_\beta \quad (31)$$

Then, the phase function is designed:

$$P(\hat{a}_1, \hat{a}_2, t_\beta) = \exp[j \frac{4\pi}{\lambda} \hat{a}_1 t_\beta^2] \exp[j \frac{4\pi}{\lambda} \hat{a}_2 t_\beta^3] \quad (32)$$

Formula (32) multiplicative formula (22) can be obtained:

$$S_{rm}(\tau, t_\beta) = Q_2 \sin c[B(\tau - \frac{f_d}{\gamma} - \frac{2R_0}{c})] \exp[-j \frac{4\pi v_0 t_\beta}{\lambda}] \times \exp[-j \frac{4\pi}{\lambda} (a_1 - \hat{a}_1) t_\beta^2] \exp[-j \frac{4\pi}{\lambda} (a_2 - \hat{a}_2) t_\beta^3] \quad (33)$$

When the estimated value is equal to the actual value, the formula (32) can be simplified to:

$$S_{rm}(\tau, t_\beta) = Q_2 \sin c[B(\tau - \frac{f_d}{\gamma} - \frac{2R_0}{c})] \exp[-j \frac{4\pi v_0 t_\beta}{\lambda}] \quad (34)$$

After steps above, the envelope of the target echo is located in the same position, and the high order phase is also eliminated. This means that the effect of RM and DS on the accumulation of target echo signal has been eliminated. Therefore, the accumulation of target echo energy can be realized by FT, as shown in the following formula (35):

$$s_{rm}(\tau, f_{i\beta}) = Q_4 \sin c \left[ B \left( \tau - \frac{f_d}{\gamma} - \frac{2R_0}{c} \right) \right] \times \sin c \left[ T \left( f_{i\beta} + \frac{2v_0}{\lambda} \right) \right] \quad (35)$$

In which  $f_{i\beta}$  is the corresponding frequency of the variable  $f_{i\beta}$  as Fourier transform,  $T = N \times T_r$  represents the total accumulation time.

#### IV. ALGORITHM COMPLEXITY ANALYSIS

Set  $N, N_r, N_v, N_{a_1}, N_{a_2}, N_\theta, N_\Omega$  represent pulse number, range unit number, searching velocity number, searching acceleration number, searching jerk number, searching number of RT angle and searching number of IFR respectively.

In this section, we will analyze the computational complexity of the proposed method. For RM correction, the complexity of multiplication operations(MO) and addition

operations (AO) is  $O(\frac{NN_r \log_2(N_r)}{2} + N^2 N_r + N_\theta N_r N)$ . For

DS correction, the complexity is  $5N_\Omega(N+1)$ . For the final FT,

the complexity is  $O(\frac{NN_r \log_2(N_r)}{2})$ . The following TABLE I

lists the comparison between the algorithm of this paper and the complexity of the GRFT method in literature [7] and the GKTGDP method in the literature [12]. If

$N=N_r=N_v=N_{a_1}=N_{a_2}=N_\theta=N_\Omega$ , the algorithm complexity of

this paper is  $O(N^3)$ , the algorithm complexity of GRFT is

$O(N^5)$ , the algorithm complexity of GKTGDP is

$O(N^4 \log_2(N))$ .

TABLE I. COMPARISON OF THE COMPLEXITY OF THE THREE ALGORITHMS

Algorithm	The Complexity of Algorithm
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The method of this paper	$\frac{NN_r \log_2(N_r)}{2} + N^2 N_r$ $+ N_\theta N_r N + 5N_\Omega(N+1)$ $+ \frac{NN_r \log_2(N_r)}{2}$
GRFT	$NN_r N_v N_{a_1} N_{a_2}$
GKTGDP	$\frac{N_{a_1} NN_r \log_2(N)}{2}$ $+ \frac{N_{a_1} N_{a_2} NN_r \log_2(N)}{2}$ $+ NN_r \log_2(N_r) + 2N^2 N_r$ $+ N_r N + NN_r N_v$

## V. EXPERIMENT SIMULATION

TABLE II shows the simulation parameters of the radar system. This section will verify the effectiveness of the algorithm by simulation experiments.

TABLE II. RADAR SYSTEM PARAMETER

Simulation parameters	parameter values
Carrier frequency	1.5 GHz
Pulse width	100 us
Bandwidth	1 MHz
Sampling frequency	2 MHz
Pulse repetition frequency	200 Hz
Accumulate pulse number	199

Suppose there is a point target in the detection scene. The initial radial distance between this target and the radar is 300KM. Initial radial velocity is 3000m/s. Acceleration is 40m/s<sup>2</sup>. Jerk is 20m/s<sup>3</sup>. Adding Gauss white noise to target echo. The SNR is -15dB.

The target echo is shown in Figure 1. The target is completely submerged in noise and can't be detected directly. Figure 2 is a two-dimensional graph after the compression of the echo pulse. It can be seen from the graph that the echo of the target has a serious RM.

Figure 3 shows the value of the Radon transformation corresponding to the different polar angles. It can be seen from the graph that there is a distinct peak region in the graph. The angle of Radon transformation corresponding to the maximum value of the transform domain is chosen as the angle estimation, and the fold factor estimation value is then calculated according to (19). Figure 4 shows the result of compensating the Doppler frequency fold factor. We can see from the figure, the RM of the target has been corrected. Compared with the KT-CPF [14] which has limited search range of Doppler fold factor, the Radon transformation has no this limitation.

After the corrected RM, the Doppler spread needs to be optimized. We use the CPF method to estimate the values of acceleration and jerk. The simulation result is that acceleration\_estimate is 40m/s<sup>2</sup> and jerk\_estimate is 19.6970m/s<sup>3</sup>. The estimated deviation of jerk is

$$\frac{20-19.6970}{20} \times 100\% = 1.52\% .$$

we construct the phase compensation function by acceleration estimation and jerk estimation to compensate the high-order phase term of the echo signal, so as to eliminate the influence of Doppler spread. The last coherent integration with Fourier transform (FT).

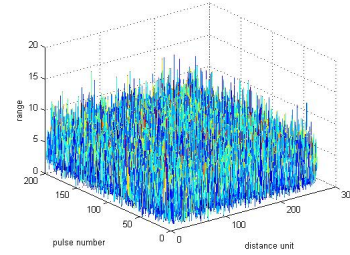


Figure 1. target echo

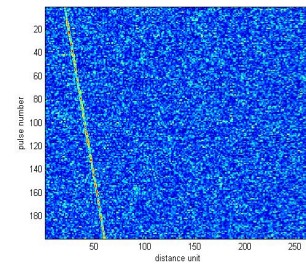


Figure 2. matched filtering result

Figure 5 shows the accumulation of the method proposed in this paper. It can be seen from the diagram that the energy of the target has been well accumulated. Figure 6 represents the result of the three order RFT algorithm. As we can see from this picture, there is a serious blind sidelobe phenomenon.

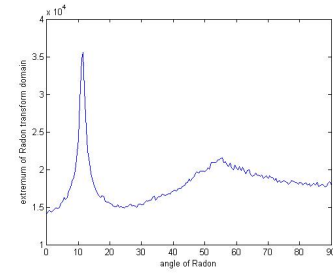


Figure 3. search results of polar angle of target

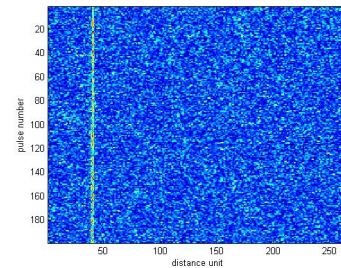


Figure 4. RM corrected result after KT-Radon

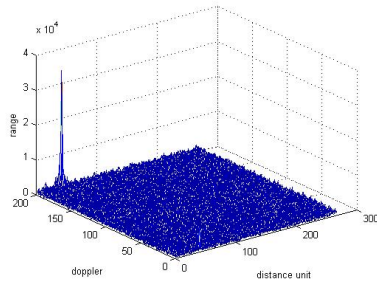


Figure 5. the accumulation result of proposed algorithm

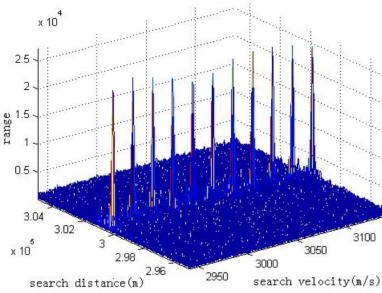


Figure 6. the accumulation result of third-order RFT algorithm

## VI. CONCLUSION

In this paper, a method of signal accumulation based on improved Keystone transform and CPF is proposed for high speed radar maneuvering target with high order motion. The method uses Keystone transform and Radon transform to correct the RM of the echo, then estimating the acceleration and the jerk and the target based on the method of CPF compensating high order phase, The last coherent integration with Fourier transform (FT). Compared with the third-order RFT algorithm, the computational complexity of the proposed method is significantly reduced, and this method is no blind speed sidelobe interference. Compared with the GKTGDP algorithm, the proposed method only needs one dimensional search for acceleration to get the acceleration and the jerk, and the complexity of the algorithm is also reduced by one order. Compared with the KT-CPF algorithm, there is no limitation on search range of Doppler fold factor in the proposed method.

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