Approximating Bistatic SAR Target Signatures with Sparse Limited Persistence Scattering Models

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Abstract—Template based Automatic Target Recognition (ATR) systems employ a large database of target signatures collected at different target-sensor geometries. In this paper, we study synthetic aperture radar (SAR) target signatures collected with a spatially separated transmitter and receiver, commonly referred to as bistatic SAR. We propose a regularized inversion method that decomposes the target into a set of scattering centers with limited persistence by utilizing the model based sparsity of scattering coefficients. The resulting sparse model can be interrogated at arbitrary target pose and bistatic geometry for online estimation of target signatures. Based on the prior work on parametric models derived for canonical reflectors, we hypothesize that the scattering coefficient as a function of the viewing angle is embedded in a low-dimensional subspace spanned by a set of functions or dictionary atoms that have a support, which is concentrated in the viewing angle domain. We utilize multivariate Gaussian functions in the bistatic angle and bisector angle domain to approximate the scattering coefficients. Furthermore, we exploit the differentiability of the observation model in the scattering center locations and the parameters associated with the basis functions to jointly estimate the scattering center locations in the continuum to avoid gridding errors and approximate the scattering coefficient as a function of the bistatic angle and the corresponding bisector. We conduct numerical simulations using a multi-bounce physical optics based electromagnetic simulator for canonical reflectors such as dihedral, trihedral and plates. We verify that the proposed numerical solution obtains a succinct representation of these objects using a sufficiently small set of scattering centers.

I. INTRODUCTION

Bistatic synthetic aperture radar (SAR) coherently integrates phase history data collected with a spatially separated transmitter-receiver pair to obtain target imagery, as shown in Fig. 1. The bistatic SAR generalizes the monostatic mode and provides diverse views of the scene. This mode of acquisition can obtain scattering behavior of specular points, which is a common scattering mechanism in flat plates. Thus, bistatic SAR provides a richer set of signatures, which might be used to improve Automatic Target Recognition (ATR) system performance.

Template based ATR systems employ a large database of target signatures collected at different target-sensor geometries to be able to identify targets at arbitrary poses. The required database size and query time increase linearly with azimuth and elevation angle coverage. For bistatic collection geometries, the required database size and query time increase exponentially as it needs to be indexed for the relative geometry of transmitter-receiver pair as well. We propose a regularized inversion method that decomposes the target into a set of scattering centers with limited persistence by utilizing the model based sparsity of scattering coefficients, generalizing the monostatic imaging setup studied in [1], [2]. The resulting sparse model can be interrogated for arbitrary target pose and bistatic geometry to the online estimation of target signatures while offering a drastically smaller database size and supporting real-time queries.

Modeling approaches such as Physical optics (PO) and Geometric theory of diffraction (GTD) [3] paved the way for analyzing high-frequency electromagnetic scattering from complex objects by decomposing them into a sparse set of dominant scattering centers such as edges, corner reflectors, wedges, discontinuities, and specular points. These dominant scattering mechanisms account for a significant fraction of the backscattered energy from a complex target [4], [5]. We adopt the flight path for the transmitter and receiver considered in the Electromagnetic (EM) simulation proposed in [6]. For a fixed bistatic angle $\theta_{\Delta} = \frac{\theta_T - \theta_R}{2}$, the transmitter and receiver move by varying the bisector of the bistatic angle $\theta_b = \frac{\theta_T + \theta_R}{2}$, where $\theta_T$ and $\theta_R$ are the azimuth angles of transmitter and receiver. We assume that the transmitter and receiver have the same elevation angle but the proposed approach is applicable for different elevation angles. The transmitter utilizes a pulse repetition rate of $2L/\lambda_c$ pulses per radian in $\theta_\theta$ domain, where $L$ is the length of the cross-range dimension and $\lambda_c$ is the wavelength of the illuminating signal. The measurement due to the scattering center at $(x_0, y_0)$ is given by

$$y(f, \theta_\Delta, \theta_b) = n(f, \theta_\Delta, \theta_b) + g(\theta_\Delta, \theta_b) \times \exp \left(-jK_{f,\phi}(x_0 \cos(\theta_b) \cos(\theta_\Delta) + y_0 \cos(\theta_\Delta) \sin(\theta_b))\right),$$

(1)

where $K_{f,\phi} = \frac{4\pi f \cos(\phi)}{c}$, $n(f, \theta_\Delta, \theta_b) \in \mathbb{C}^2 \{0, \sigma_n^2\}$ is the zero mean complex Gaussian noise of variance $\sigma_n^2$, $g(\theta_\Delta, \theta_b) \in \mathbb{C}^2$ is the multivariate scattering coefficient of the scene as a function of the bistatic angle $\theta_\Delta$ and the azimuth angle $\theta_b$ of the bisector of the bistatic angle. We extend the function approximation approach proposed in the monostatic case to recover the scattering coefficients from the measurements (1).

We parametrize the scattering coefficients as a function of the bistatic angle and the azimuth angle of the bistatic angle’s bisector. Using this parameterization, we estimate the reflectors in the scene along with the scattering coefficients using phase-history measurements generated from models of standard reflectors such as rectangular plates, dihedral, and trihedral based on the multi-bounce physical optics integration (PO) [7]. We empirically evaluate the approximation error of
the proposed algorithm through numerical studies. We note that the proposed approach can successfully recover the underlying scattering mechanism using a limited set of measurements, which can lead to a substantial reduction in the computational burden on commonly used EM simulators.

II. SYSTEM MODEL

We consider a spatially separated transmitter and a receiver located at a distance of $R_t$ and $R_r$ from the origin with the common angle of elevation $\phi$ and an azimuth angle of $\theta_t$ and $\theta_r$, respectively. The scene reflectivity with $K$ scattering centers expressed as a function of the spatial locations $(x_i, y_i)$, and the angles $\theta_b, \theta_\Delta$ and $\phi$ is given by

$$g(x, y, \theta_b, \theta_\Delta, \phi) = \sum_{i=1}^{K} \delta(x - x_i, y - y_i) h_i(\theta_b, \theta_\Delta, \phi),$$

where $\delta(x - x_i, y - y_i)$ is the Dirac delta generalized function indicating the location of the $i^{th}$ scattering center, and $h_i(\theta_b, \theta_\Delta, \phi)$ is the scattering coefficient as a function of the bistatic angle $\theta_\Delta$, and the bisector angle or pose $\theta_b$ for a fixed elevation $\phi$. The phase history measurements can be obtained as follows

$$r(f_m, \theta_b, \theta_\Delta, \phi) = n(f_m, \theta_b, \theta_\Delta, \phi) + \sum_{i=1}^{K} h_i(\theta_b, \theta_\Delta, \phi) \exp(-jK_{f_m, \phi}(x_i \cos(\theta_b) \cos(\theta_\Delta) + y_i \cos(\theta_\Delta) \sin(\theta_b))),$$

where $K_{f_m, \phi} = \frac{4\pi f_m \cos(\phi)}{c}$, $f_m = f_c + \frac{(m-L_B/c)c}{2L}$, $m = 0, \cdots, M - 1$, $M$ is the number of samples per pulse, $n(f_m, \theta_b, \theta_\Delta, \phi)$ is the additive white Gaussian receiver noise with variance $\sigma_n^2$. The transmitter and receiver trace a path where the radar platform transmits $N_p$ pulses by varying the pose of the system denoted by the set $\Theta_b = \{\theta_b^1, \theta_b^2, \ldots, \theta_b^{N_p}\}$ for each value of the bistatic angle $\theta_\Delta$. This procedure is repeated for $N_\Delta$ values of the bistatic angle in the set denoted by $\Theta_\Delta = \{\theta_\Delta^1, \theta_\Delta^2, \ldots, \theta_\Delta^{N_\Delta}\}$. The total number of pulses collected through this procedure is $N_p N_\Delta$. Next, we study the scattering mechanisms of certain canonical reflectors.

A. Bistatic scattering phenomenon

For the case of flat rectangular plate of width $W$ and height $H$, the scattering behavior has been studied in [8]. The reflectivity of a flat rectangular plate with the surface normal at an orientation of $\phi = 0, \theta = 0$ in the elevation and azimuth direction with respect to the origin, and with the transmitter and receiver at an orientation of $\theta_t, \theta_r$ in azimuth domain and $\phi$ in elevation domain is given as

$$h_p(\theta_t, \theta_r, \phi; \bar{\theta}, \bar{\phi}) = A \text{sinc}(k H \cos(\phi)) \times \frac{sinc}{2} \left(\frac{k W}{2} \cos(\phi) \left(\sin(\theta_t) + \sin(\theta_r)\right)\right),$$

for $\theta_t, \theta_r, \phi \in [-\pi/2, \pi/2]$, where $A = 2\pi W H f_c / c$. The scattering coefficient for a rectangular plate at an arbitrary orientation $\bar{\phi}, \bar{\theta}$ is obtained by rotating the coordinate system of the transmitter and receiver such that it aligns with the reflector [8]. We see that if the bisector of the bistatic angle in the azimuth domain aligns with the normal of the rectangular plate, the response is maximum irrespective of the bistatic angle. Similarly, the scattering function of a dihedral of width $W$ and height $H$ located at a canonical orientation $(\theta = 0, \phi = 0)$ is given by

$$h_d(\theta_t, \theta_r, \phi; \bar{\theta}, \bar{\phi}) = A \text{sinc}(k H \cos(\phi)) \times \frac{sinc}{2} \left(\frac{k W}{2} \cos(\phi) \left(\sin(\theta_t) + \sin(\theta_r)\right)\right) \times \begin{cases} \sin(\phi), & \phi \in [0, \pi/4] \\ \cos(\phi), & \phi \in [\pi/4, \pi/2], \end{cases}$$

where $A = 4\pi W H f_c / c$ and $\theta_t, \theta_r \in [-\pi/2, \pi/2]$. The scattering behavior is similar to that of a plate except for the amplitude level. Finally, the scattering function of a trihedral of height $H$ located at a canonical orientation $(\theta = 0, \phi = 0)$ is given by

$$h_t(\theta_t, \theta_r, \phi; \bar{\theta}, \bar{\phi}) = 0.5 A \text{sinc}(k H \cos(\phi)) \times \frac{sinc}{2} \left(\frac{k W}{2} \cos(\phi) \left(\sin(\theta_t - \pi/4) - \cos(\theta_r - \pi/4)\right)\right) \times \begin{cases} \sin(\phi + \pi/4 - \gamma), & \phi \in [0, \gamma] \\ \cos(\phi + \pi/4 - \gamma), & \phi \in [\gamma, \pi/2] \\ + \sin(k H \cos(\phi) \left(\cos(\theta_t + \pi/4) - \cos(\theta_r + \pi/4)\right)) \times \begin{cases} - \cos((\theta_t + \theta_r)/2 - \pi/4), & \theta_t \in [-\pi/4, 0] \\ \sin((\theta_t + \theta_r)/2 - \pi/4), & \theta_t \in [0, \pi/4], \end{cases} \end{cases}$$

$$A = 4\sqrt{3}\pi H^2 f_c / c,$$
Motivated by the strategy proposed in the monostatic case for approximating the functions using appropriately chosen basis functions [2], we extend this technique to the bistatic configuration. In Section II-A, we discussed the models for the scattering coefficients of certain canonical reflectors such as rectangular plates, dihedral, and trihedral. Next, we extend the approximation method used in this work.

### B. Approximation using Gaussian functions

The scattering coefficients of the reflectors that we considered have limited persistence in the bistatic angle and the pose and a periodic structure due to the circular nature of illumination. Based on the analytical models, we hypothesize that these functions belong to a subspace of shift-invariant functions with parameters that are controlled by the dimensions of the object. To be more specific, suppose $h(\theta_b, \theta_D; \phi, D)$ denote the reflector’s response, which has the canonical orientation of $\theta = 0$, $\phi = 0$, and $D$ is the set of parameters such as length, height and type of reflector. The response of the same reflector with an arbitrary orientation $\theta, \phi$ is assumed to be $h(\theta_b - \theta, \theta_D; \phi - \phi, D)$. We further assume that the functions described in the Section II-A are smooth and have a nearly bandlimited frequency spectrum. Using the assumptions stated above along with the fact that these functions have a limited support in $\theta_b$ and $\theta_D$ domain owing to the limited persistence, we propose that these functions have a sparse representation in the set of multivariate Gaussian functions parametrized by the center and the width of the Gaussian function by using the results presented in [9], [10] for function approximation and interpolation using Gaussian kernels. We utilize the following multivariate Gaussian functions in the bistatic angle $\theta_D$ and the pose denoted by $\theta_b$ with a structure as shown below

$$\tilde{\Psi}(\Lambda; \theta_b, \theta_D) = \exp \left( - \frac{(\theta_b - \mu_b)^2}{2\sigma_b^2} - \frac{(\theta_D - \mu_D)^2}{2\sigma_D^2} \right). \quad (7)$$

where $\Lambda = \{\mu_b, \mu_D, \sigma_b, \sigma_D\}, \{\mu_b, \sigma_b\}$, and $\{\mu_D, \sigma_D\}$ are the center and width of the Gaussian function in $\theta_b$ and $\theta_D$ domain, respectively. The scattering coefficients are given by

$$h_i(\theta_b, \theta_D; \phi) = \int \tilde{\Psi}(\Lambda; \theta_b, \theta_D) d\nu_1(\Lambda)$$

$$= \sum_j c_j \int \tilde{\Psi}(\Lambda; \theta_b, \theta_D) \delta(\mu_b - \hat{\mu}_b(j), \mu_D - \hat{\mu}_D(j)) \times \delta(\sigma_b - \hat{\sigma}_b(j), \sigma_D - \hat{\sigma}_D(j)) d\mu_b d\mu_D d\sigma_b d\sigma_D \delta(\theta_b, \theta_D; \phi)$$

$$= \sum_j c_j \tilde{\Psi}(\hat{\mu}_b(j), \hat{\mu}_D(j), \hat{\sigma}_b(j), \hat{\sigma}_D(j); \theta_b, \theta_D)$$

such that the number of points in the support set of the discrete measure $\nu_1$ is bounded and the approximation error $\epsilon$ is negligible, where $c_j \in \mathbb{C}$ is the complex weight of each point in the support set. We utilized the property that a multi-dimensional generalized Dirac function is a tensor-product of 1-D Dirac functions, therefore the function is separable. In the next section, we formulate the problem and present the algorithm to recover the characteristics of the reflectors in the scene.

### III. Problem Statement

We consider the problem of estimating the locations and the scattering coefficients of the reflectors in the scene using phase history measurements in (3). We assume that the scene contains $K$ scattering coefficients and utilize the Gaussian approximation as shown below

$$r(f_m, \phi, \theta_D, \phi) = \sum_{i=1}^{K} h_i(\theta_b, \theta_D, \phi) \int \delta(x - x_i, y - y_i)$$

$$\exp(-jK(x \cos \theta_b + y \sin \theta_b)) \, dx \, dy$$

$$+ n(f_m, \theta_b, \theta_D, \phi)$$

$$r(f_m, \theta_b, \theta_D, \phi) = \int \tilde{\Psi}(\Omega; \theta_b, \theta_D, f_m) d\nu(\Omega)$$

$$+ n(f_m, \theta_b, \theta_D, \phi)$$

$$\exp(-jK(x \cos \theta_b + y \cos \theta_b \sin \theta_b))$$

where $\tilde{\Psi}(\Lambda; \theta_b, \theta_D)$ is the angular component of the measurement operator given in (7), $\Omega$ is the product space of spatial coordinates $(x, y)$, and the center and width of the Gaussian functions in azimuth domain $\Lambda = \{\mu_b, \mu_D, \sigma_b, \sigma_D\}$

$$\nu_1(\mu_b, \mu_D, \sigma_b, \sigma_D; x, y) = \sum_j c_j \delta(\mu_b(k, x, y))$$

$$\delta(\mu_D - \hat{\mu}_D(k, x, y), \sigma_b - \hat{\sigma}_b(k, x, y), \sigma_D - \hat{\sigma}_D(k, x, y)),$$

$$\nu(x, y, \mu_b, \mu_D, \sigma_b, \sigma_D) = \sum_k c_k \delta(x - x_k, y - y_k)$$

$$\delta(\mu_b - \hat{\mu}_b(k), \mu_D - \hat{\mu}_D(k), \sigma_b - \hat{\sigma}_b(k), \sigma_D - \hat{\sigma}_D(k))$$

where $\nu_1(\mu_b, \mu_D, \sigma_b, \sigma_D; x, y)$ is the conditional measure on the domain of scattering coefficient conditioned on the spatial location of the scattering center, and $\nu(x, y, \mu_b, \mu_D, \sigma_b, \sigma_D)$ is the product measure. We formulate the estimation problem as follows

$$\min_{\nu} \left\| r - \int \tilde{\Psi}(\Omega; f, \theta_b, \theta_D) d\nu(\Omega) \right\|_2^2$$

subject to $\|\nu\|_{TV} \leq \tau$,

where $f \in \mathbb{R}^M$ are the set of frequencies used, $r \in \mathbb{C}^{MN_\theta N_\phi}$ are the phase history measurements, $\|\nu\|_{TV} = \sum_k |c_k|$ is the total variation norm on the set of discrete measures, $c_k \in \mathbb{C}$, $\int_{\Omega} : \mathbb{C}^{MN_\theta N_\phi} \to \mathbb{C}^{MN_\theta N_\phi}$ is the vector valued integral, and $\tau$ is the constraint on the total variation norm for the discrete measure that limits the size of the support set of the discrete measure. The problem stated in (10) is a convex relaxation of the problem with the constraint on the size of the support set of the measure given by $|\text{supp}(\nu)| \leq K$. The total-variation norm [11] is equivalent to the $\ell_1$ norm if the space $\Omega$ is discrete. Discretization of the 6-d space $\Omega$ leads to a steep increase in the computational cost, therefore we solve this problem in the continuum using a property of the measurement operator $\Psi$. Since the function $\tilde{\Psi}(x, y, \mu_b, \mu_D, \sigma_b, \sigma_D; \theta_b, \theta_D, \phi) \in \mathbb{C}^{MN_\theta N_\phi}$ is differentiable in the unknown parameters, we utilize the method proposed in [12] to solve the problem. We solve the problem in (10) using the alternating minimization procedure to estimate the
scattering centers and the scattering coefficients expressed using Gaussian functions. We sequentially add points to the support set until the convergence condition, which is a function of the constraint \( \tau \) is satisfied or until the size of the support set exceeds an upper bound. In the next section, we present the simulation results of the proposed recovery step.

IV. NUMERICAL SIMULATIONS

For this section, we generate the phase history measurements using the iterative physical optics based EM simulation [7]. We used the parameters for generating the phase history measurements as follows: step size bisector \( \Delta \theta_b = 0.04 \) degrees, \( \theta_\Delta = [0, 1, 2, 3, 4, 5, 10, 20, 30, 90] \), elevation \( \phi = 12 \) degrees, total variation norm bound \( \tau = 20 \), center frequency \( f_c = 9.9 \times 10^8 \) Hz. We perform the simulation with trihedral and dihedral in one case and two rectangular plates of different dimensions in the other setup. We obtain the phase history measurements over a subset of \( \theta_\Delta \) and vary the pose \( \theta_b \in [0, 90] \) in the case of trihedral and dihedral, and \( \theta_b \in [-50, 50] \) for the setup with plates. The solution obtained using the method presented in section III is the representation of the scene using scattering centers on the entire range of the viewing angles \( \Theta_b \times \Theta_\Delta \) given by

\[
r(f, \theta_b, \theta_\Delta) = \sum_k \hat{c}_k \Psi(L_k, x_k, y_k; f, \theta_b, \theta_\Delta). \tag{11}
\]

Each estimated scattering center located at \( \{x_i, y_i\} \) shown in Figs 2(a) and 3(a) has the scattering response given by a multivariate Gaussian with mean \( \{\mu_k, \mu_\Delta\} \), and width \( \{\sigma_k, \sigma_\Delta\} \). Fig. 2 and 3 show the reconstruction for the simulation setup with two plates, and trihedral-dihedral, respectively. We compare the reconstruction of each of these two objects in the scene with the true responses of the individual objects. The scattering response for a reflector is computed by coherently adding the response of the scattering centers that lie within the true boundary of the object. We observe that the scattering response in the \( \Theta_b \) domain is approximated with low approximation error, while there is some distortion in \( \Theta_\Delta \) domain. We also show that the approximation error for the special case of monostatic setup is negligible in Fig. 2(f), 2(g), 3(f), and 3(g).

Next, we utilize the back-projection method proposed in [13] to generate the images using the phase-history measurements, which serves as a ground truth, and generate the images using the estimated model. The figures show that the estimated model captures the scattering mechanisms of the plates for different configurations of the bistatic angle \( \theta_{\Delta} \) and the pose \( \theta_b \). Similarly, we form the images using the back-projection method for both the phase-history measurements and the estimated model for the trihedral and dihedral setup.

The images in Fig. 6, and 7 show that the estimated models successfully capture the scattering mechanism of trihedrals and dihedrals.

V. CONCLUSION

We presented a method that compresses bistatic SAR signatures into a set of limited persistence scattering primitives with response defined over the bistatic and the pose angles of the scene center. We empirically verified the performance of the proposed approach on models based on the multi-bounce geometrical optics and physical optics integration as well as a simulation that utilizes iterative physical optics to compute the scattered energy. A potential direction for future work is to integrate the online signature prediction capability of the proposed modeling strategy with a template based ATR system and validate its performance.

REFERENCES

Fig. 2. Fig. 2(a) shows the detected support that represent the reflectors. Figs. 2(b), and 2(d) show the true response of the smaller plate $P1$ and larger plate $P2$ as a function of bisector angle $\theta_b$ and the bistatic angle $\theta_\Delta$ at the center frequency. Figs. 2(c), and 2(e) show the reconstructed response of the smaller plate $P1$ and larger plate $P2$ as a function of bisector angle $\theta_b$ and the bistatic angle $\theta_\Delta$ at the center frequency. The reconstructed response of the plates overlaid with the ground truth as a function $\theta_b$ for the monostatic case of $\theta_\Delta = 0$, which is a subset of the results are shown in Fig. 2(f) and Fig. 2(g), respectively.

Fig. 3. Fig. 3(a) shows the detected support points that represent the reflectors. Fig. 3(b) and 3(d) show the true response of the trihedral and dihedral as a function of bisector angle $\theta_b$ and the bistatic angle $\theta_\Delta$ at the center frequency, respectively. Figs. 3(c) and 3(e) show the reconstructed response of the trihedral and dihedral as a function of bisector angle $\theta_b$ and the bistatic angle $\theta_\Delta$ at the center frequency. The reconstructed response of the dihedral and trihedral overlaid with the ground truth as a function $\theta_b$ for the monostatic case of $\theta_\Delta = 0$ are shown in Fig. 3(f) and Fig. 3(g), respectively.
The bistatic angle used in the simulation is $\theta_b = 5$, and the bisector angle is varied in the interval $\theta_b \in [-15, -10]$, and $\theta_b \in [-2.5, 2.5]$, respectively.

The bistatic angle used in the simulation is $\theta_b = 5$, and the bisector angle is varied in the interval $\theta_b \in [10, 15]$, and $\theta_b \in [20, 25]$, respectively.

The bistatic angle used in the simulation is $\theta_b = 5$, and the bisector angle is varied in the interval $\theta_b \in [0, 10]$, and $\theta_b \in [20, 30]$, respectively.

The bistatic angle used in the simulation is $\theta_b = 5$, and the bisector angle is varied in the interval $\theta_b \in [40, 50]$, and $\theta_b \in [50, 60]$, respectively.