Artificial Neural Networks to Solve Doppler Ambiguities in Pulsed Radars

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***Abstract*—Undersampling is a common issue in pulsed radars. It causes incorrect estimates of the velocity of detected targets due to the Doppler frequency ambiguity. In this paper, we propose a novel way to estimate the Doppler frequency by using Artificial Neural Network (ANN). The results demonstrate that this new method has a better performance and lower computational cost than the traditional methods like Chinese Remainder Theorem and Robust Chinese Remainder Theorem.**

I. Introduction

An important task of pulsed Doppler radars is the determination of the Doppler frequency of the received signal coming from moving targets. Depending on the Pulse Repetition Frequency (PRF) and the target's radial velocity, an ambiguous estimate can be produced. This occurs when the received signal is subsampled by a PRF lower than the Doppler frequency related to the target's velocity.

A subsampled waveform produces frequency measurements that are the remainders of the division of the sampling frequency (PRF) by the real waveform frequency (Doppler). In order to find the unfolded frequency, a very common approach is to sample the waveform with multiples PRFs and use the measured frequencies as input to the Chinese Remainder Theorem (CRT) [1, 2, 3]. With a simple and straightforward procedure, it is possible to obtain the unambiguous Doppler frequency.

Although the CRT is quite accurate and fast, it is also extremely fragile when the remainders present deviations due to noise or any other reason. In order to alleviate this problem, a Robust Chinese Remainder Theorem (RCRT) was proposed in [4, 5] that guarantees a frequency reconstruction with an error limited by the maximum measurement remainder error. Unlike the original CRT, the RCRT has an elevated computational cost [6].

In [7] an algorithm based on CRT is used to generate data, which are used to design an Artificial Neural Network (ANN) to solve the ambiguity problem. However, because the ANN is based on a CRT, it is not capable of solving problems when there is noisy input.

In this paper we use an ANN to emulate the behavior of the RCRT over a pulsed radar set up. So this approach is more robust than [7] and can solve problems in the presence of noise at the system input. A overview of our method can be seen in Fig.1. As far as we are aware, this is the first time that a machine learning tool is used for emulating a RCRT.

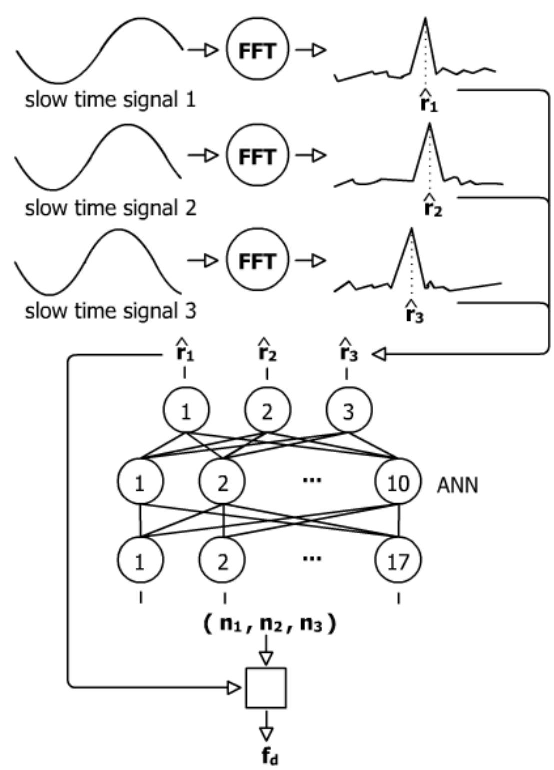

Figure 1: Overview of the method to determine the Doppler frequency using ANN

The remaining work is organized as follows. In section II, we present the Problem Statement. In section III, we describe the RCRT in more details. In section IV, the method using ANN is presented. In section V, we present numerical results. And in section VI, we finally conclude the work with some considerations.

II. Problem Statement

Set a pulsed Doppler radar system, the received signal $x[n]$ of a detected moving target, sampled in slow time during a period T, is a complex sine wave with additive noise and submitted to a sampling frequency (F_s) equal the radar's Pulse Repetition Frequency (PRF), F_s = PRF. The sampling time (t_s) is $t_s = 1/F_s$ and the total number of samples $M = F_s \cdot T$. So, $x[n]$ can be written as follows:

$$\begin{aligned} x[n] &= s[n] + w[n] \\ &= a \exp\left(j2\pi f_d t_s n\right) + w[n], \quad 0 \leq n \leq M-1, \end{aligned} \tag{1}$$

where $s[n]$ and $w[n]$ are, respectively, the discrete complex sine wave and the discrete additive Gaussian white noise, f_d is the $s[n]$ Doppler frequency and a is the signal amplitude.

The Discrete Fourier Transform of the signal $x[n]$ is given by:

$$\begin{aligned} X[k] &= \mathscr{F}\{x[n]\} \\ &= \sum_{n=0}^{M-1} x[n] \cdot \exp(-j2\pi kn/M), \quad 0 \leq k \leq M-1. \end{aligned} \tag{2}$$

The discrete signal in frequency domain $X[k]$ allows us to obtain an estimative $\hat{f}_d$ of the Doppler frequency f_d, in (1). This estimative is achieved by searching k where the absolute value of $X[k]$ is maximum; if $F_s > f_d$, $\hat{f}_d = k/T$. This means the waveform is oversampled and the solution is trivial.

However, if $F_s < f_d$, a close estimate of $\hat{f}_d$ can't be found directly because the maximum frequency Doppler measurable is less than F_s. In fact, due to the periodicity of the discrete Fourier transform, what can be measured is the remainder f_r of the division of f_d by F_s, such as:

$$f_d \equiv f_r \pmod{F_s}. \tag{3}$$

The CRT is a common approach to estimate $\hat{f}_d$ in a subsampled system, with L distinct F_{si}, where $F_{si} < f_d$, $\forall i \in [1, L]$. In this way, $\hat{f}_d$ is reconstructed from L remainders f_{ri} modulo L distinct frequencies F_{si}, ($L \geq 2$). In CRT all the F_s are co-prime. In RCRT method all the F_{si} have a greatest common divisor G greater than 1 that allows some deviations in the remainders due to noise.

These deviations in the remainders implies that there are a selection of sets of L remainders measurements $\hat{f}_{r_i}$ that allows correct reconstruction of the Doppler frequency in contrast to the CRT method, where each set of L remainders unfolds a unique value of Doppler frequency.

III. The Robust Chinese Remainder Theorem

The RCRT is described in details in [5]. It is not scope of this paper present the RCRT. For now, what we need to understand is the basic conditions in the algorithm derived from the RCRT and its inputs and outputs. This way, we might generate the dataset that will allow us to train and validate the ANN. The RCRT algorithm works on the data extracted from the sampled spectrum $X[k]$ (2), so the equation (3) becomes:

$$r_u \equiv r_i \pmod{M_i}, \qquad 1 \leq i \leq L, \tag{4}$$

where $M_i = F_{s_i} \cdot T$ is the number of samples, $r_i = f_{r_i} \cdot T$ is the remainder and is also equal k where the absolute value of $X[k]$ is maximum. Finally, $r_u = f_d \cdot T$ is the unfolded remainder from L remainders.

From (4), it is not difficult to see that r_u can be obtained in the following way:

$$r_u = \frac{1}{L} \sum_{i=1}^{L} (n_i M_i + r_i), \tag{5}$$

where n_i are the unknown integers that unfolds r_u.

In this scenario, we will assume that the remainders might have errors, so:

$$0 \leq \hat{r}_i \leq M_i \qquad \text{and} \qquad \forall i \in [1, L], |\hat{r}_i - r_i| \leq \tau, \tag{6}$$

where τ is the maximal error level, called remainder error bound. We now want to reconstruct r_u from these erroneous remainders $\hat{r}_i$ and the known moduli M_i. With these erroneous remainders, equation (5) becomes:

$$\begin{aligned} r_u &= \frac{1}{L} \sum_{i=1}^{L} (\hat{n}_i M_i + \hat{r}_i) \\ &= \frac{1}{L} \sum_{i=1}^{L} (\hat{n}_i M_i + r_i + \Delta r_i), \end{aligned} \tag{7}$$

where $\hat{n}_i$ are unknown and $\Delta r_i = \hat{r}_i - r_i$ denote the errors of the remainders. From (6), $|\Delta r_i| \leq \tau$. The basic idea of the RCRT is to accurately determine the unknown integers $\hat{n}_i$ in (7) which are the folding integers that may cause large errors in the reconstructions if they are not correct.

To achieve that, the RCRT states two conditions. The first one says, once $M_i = G \cdot \Gamma_i$, $1 \leq i \leq L$, where Γ_i are pairwisely co-prime and G is the greatest common divisor of M_i, r_u must be lesser than the least common multiplier of M_i. That is:

$$G = \gcd(M_1 M_2 \cdots M_L); \tag{8}$$

$$M_i = G \cdot \Gamma_i, \quad \forall i \in [1, L]; \text{ and} \tag{9}$$

$$0 \leq r_u \leq \text{lcm}(M_1 M_2 \cdots M_L) = \frac{1}{G^{L-1}} \prod_{i=1}^{L} M_i. \tag{10}$$

And the second condition is:

$$G > 4\tau. \tag{11}$$

Fulfilled the conditions (10) and (11), the RCRT ensures that conducting a sweep on $(\hat{n}_1, \hat{n}_i)$ that minimizes (12), for $2 \leq i \leq L$, all correct n_i can be found.

$$\text{minimum} = |\hat{n}_1 M_1 + \hat{r}_1 - \hat{n}_i M_i - \hat{r}_i|. \tag{12}$$

From (6) and (11) we have:

$$\forall i \in [1, L], |\hat{r}_i - r_i| < \frac{G}{4}. \quad (13)$$

And, finally, considering in (12) all correct n_i and (13):

$$|\Delta r_1 - \Delta r_i| < \frac{G}{2}, \qquad 2 \leq i \leq L. \quad (14)$$

From condition (14) we can generate the ANN's dataset. To do that we choose all combinations $\hat{r}_i$, $1 \leq i \leq L$ that satisfy (14); this will be the input to the ANN. Using this input into the RCRT algorithm, we have our output n_i, $1 \leq i \leq L$, to the ANN.

IV. ANN BASED ON RCRT TO SOLVE DOPPLER AMBIGUITY

Artificial Neural Networks (ANN) [8] have been largely employed over the last decades. They are a nonlinear statistical machine learning technique inspired by biological neural networks, widely used for pattern recognition. The purpose of a neural network is to map an input into a desired output [9], which is also called as label.

A feedforward network is composed of neurons arranged in layers. Data are introduced into the system through an input layer. This is followed by processing in one or more intermediate (hidden) layers. Output data emerge from the network's final layer. The objective of the learning algorithms for ANNs is to adjust the weights on all the edges. Within each neuron, the weighted inputs from the previous layer are then combined and go through an activation function, which traditionally constrains its output into [0;1] . Sigmoid functions are widely used for this purpose, although other functions are possible.

To obtain the set of weights we need to train the ANN. We use two dataset during training: training data and validation data. The validation dataset is used during training to decide when to stop the training (i.e. when the error on the validation set starts increasing, which is a sure sign of overfitting). The test set is used after training, to evaluate the performance of our model.

Knowing the restraints and working conditions of the RCRT, we can define a radar setup and obtain deterministically all the possible inputs and outputs of these into RCRT that allows the correct Doppler frequency reconstruction. These will fill the ANN's dataset which is used to train the ANN.

A. Radar set up and ANN's dataset

To generate the ANN's dataset, we considered the maximum radial velocity detection to be $v_{max} = 1,200m/s$. Assuming a radar system with transmission carrier frequency $f_c = 1,5GHz$, it has approximately maximum Doppler frequency detection $f_d = 12,000Hz$.

We choose working with three PRF, which are $F_{s_1} = 1,600Hz$, $F_{s_2} = 1,920Hz$, $F_{s_3} = 2,240Hz$, and thus $L = 3$. Adopting a sampling period $T = 0.05s$, we have the number of samples $M_1 = 80$, $M_2 = 96$, $M_3 = 112$ (9), and, therefore, $G = 16$ (8), $\Gamma_1 = 5$, $\Gamma_2 = 6$, $\Gamma_3 = 7$ (9).

Now, for each measurable frequency by this setup until the defined maximum Doppler frequency detection, we generate all possible triplet $\hat{r}_1, \hat{r}_2, \hat{r}_3$ that satisfy (14) and use as input to a previously implemented RCRT algorithm. This covers all SNR scenario. The expected output are the triplet n_1, n_2, n_3 that correctly reconstructs the Doppler frequency unfolded remainder r_u in (7).

Using the input and output triplets we can construct the ANN's dataset. The complete dataset has over $400,000$ (four hundred thousand) examples. We present an extract of the ANN's dataset in Table I.

Table I: Extract of the ANN's dataset

Input triplet ($\hat{r}_1, \hat{r}_2, \hat{r}_3$)			Output triplet (n_1, n_2, n_3)		
00	01	03	0	0	0
19	16	14	0	0	0
29	29	27	0	0	0
55	50	48	0	0	0
73	75	77	0	0	0
67	22	84	3	3	2
73	23	82	3	3	2
69	25	83	3	3	2
74	24	83	3	3	2
76	26	91	3	3	2

B. ANN Configuration

We use $\hat{r}_1$, $\hat{r}_2$ and $\hat{r}_3$ as input of our ANN. There are 17 different outputs of n_1, n_2 and n_3. These are used as output of the ANN and are presented in Table II.

Table II: ANN Output

Output triplet (n_1, n_2, n_3)			ANN Output
0	0	0	01
1	0	0	02
1	1	0	03
1	1	1	04
2	1	1	05
2	2	1	06
2	2	2	07
3	2	2	08
3	3	2	09
4	3	2	10
4	3	3	11
4	4	3	12
5	4	3	13
5	4	4	14
6	5	4	15
7	5	5	16
7	6	5	17

In this work we use an ANN with 3 neurons in input layer ($\hat{r}_1, \hat{r}_2, \hat{r}_3$), 10 neurons in hidden layer and 17 neurons in output layer (Table II), as seen in Figure 2.

C. ANN Training

The dataset used to configure a feed-forward back propagation ANN consists of 415,000 examples. It was randomly divided into 3 parts: 60% was used to training, 10% used to validation and 30% of unseen examples were used to test. We used 10-fold cross validation. The function *train* from Matlab

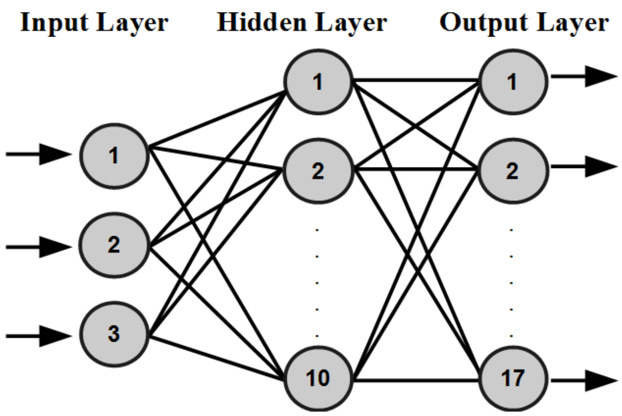

Figure 2: The 3-10-17 ANN used in this work

was used to train the ANN, considering the parameters of Table III.

The cross entropy error performances of training, testing and validation process for different numbers of epochs are shown in the Figure 3. The error at epoch 199 is 6.22x10^{-7}. All examples of test dataset (unseen examples) were correctly classified.

Table III: Parameters of function *train* (Matlab)

Train function variables (MATLAB)	Parameters used
trainFcn	trainscg
performFcn	crossentropy
divideMode	sample
stop	Validation stop

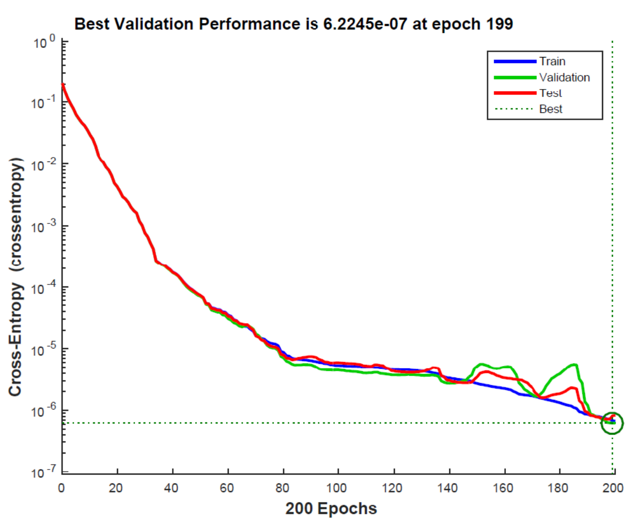

Figure 3: Performance of the 3-10-17 ANN. Blue: train, green: validation and red: test. The best validation performance is 6.22x10^{-7} at epoch 199.

There are other types of multiclass classifiers (i.e. Support Vector Machines), and other variations from ANN that have been used over past years, for example, Convolutional Neural Nets (CNN) [10], which normally has a better performance than a ANN. However, in this work the results presented in section IV demonstrate the a ANN is so robust to ensure a high classification perfomance that it is not necessary to use other types of classifiers.

V. Results

To conduct a comparative simulation between the behavior of the ANN trained with the RCRT input and output data and the RCRT itself, it was used a whole new set of data, different from the one used in the ANN's dataset. We generated waveforms like the one described in (1) adopting as Doppler frequencies values spaced out $1,000Hz$ from the minimum Doppler frequency equal $\frac{1}{T} = 20Hz$, until the maximum $12,000Hz$. It totals 12 frequencies tested.

Each one these waveforms are sampled with the three PRF defined in section IV.A and summited to a Discrete Fourier Transform. This way we can extract the three remainders measures $\hat{r}_1, \hat{r}_2, \hat{r}_3$ for each Doppler frequency in a specific SNR. It is done 2.500 times. We varies the SNR from $-25dB$ to $0dB$ spaced out $0,5dB$. It totals 51 different SNR.

Thus, for each SNR we have 2,500 experiments in 12 frequencies, which leads to 30,000 trial per SNR. The whole experiments has 1,530,000 trials.

We used the three remainders measures $\hat{r}_1, \hat{r}_2, \hat{r}_3$ into the RCRT algorithm and into the trained ANN and compared the respective outputs with the theoretical expected answer. When we have a match we register a success, otherwise, a failure. In the specific case of the ANN, all the outputs has associated a degree of certainty and only when it exceeds 90% we consider the result for a success. Obviously when the degree of certainty is below 90%, it is automatically a failure.

The total of success are divided for the total of trials per SNR and the empirical Probability of determination of correct Doppler frequency (Pd) x SNR are plotted in the Figure 4:

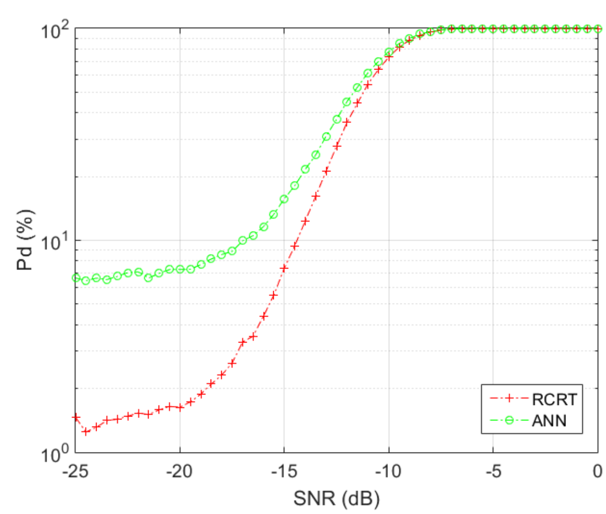

Figure 4: Comparison between the empirical P_d for the Robust CRT and ANN. In lower SNR the ANN performance is better than the RCRT.

As we can see, the ANN has a better performance than the RCRT when the SNR is low. The ANN was trained with data errors, due to low SNR, until the limit established in (14). Was not considered if any of the input terms $(\hat{r}_1, \hat{r}_2, \hat{r}_3)$ surpass the limit. So we understand that when some of the triplet terms presents a error greater than excepted, the ANN yet manages to associate the input to a correct output with degree of certainty above 90%. This is different of would happen with the use o RCRT. Both methods converge to a equivalent performance when the SNR is higher, which is expected because lesser random inputs.

Considering the computational cost, had been shown in [6] that the RCRT requires $L-1$ two-dimensional searching, more exactly:

$$\text{number of searches} = \gamma_1 \sum_{i=2}^{L} \gamma_i, \tag{15}$$

where:

$$\gamma_i \triangleq \prod_{k=1}^{L} \frac{\Gamma_k}{\Gamma_i}. \tag{16}$$

So, the number of searches has an order of $(L-1)\Gamma_i^{2(L-1)}$.

In comparison, once the ANN is trained, the computational cost of its practical use has an order of some mathematical operations, what is, in fact, a faster approach.

VI. Conclusion

In this paper a novel way of determining Doppler frequency by using ANN based on RCRT was proposed.

Different from method proposed by [7], our approach is able to find the correct Doppler frequency even when there is noise at system's input.

The results presented in section V demonstrate that our approach is superior in performance and computational cost to the RCRT algorithm.

We believe that the problem we described in this paper is generic in signal processing and our method will have more applications in other areas.

To solve the ambiguity problems normally the CRT or RCRT algorithms use just a single dimensional approach (range or Doppler frequency). Future works intend to solve ambiguity problems considering at same time range or Doppler frequency by using ANN.

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