Comparison of Bi-Modal Coherent Sea Clutter Models

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Abstract—Modelling and simulation of radar sea-clutter is extremely important for evaluating radar detection algorithms and to stimulate radar processors during development and testing. Of the many aspects of sea-clutter, the Doppler spectrum plays a key role in determining the statistics which are important for coherent processing. In this paper, the evolving Doppler spectrum model and the spatially and temporally limited clutter model are described and analysed. Simulated data is generated using both models with parameters estimated from the Ingara medium grazing angle sea-clutter data set. This allows a comprehensive study of the strengths and weaknesses of each model over a range of polarisations and swell angles.

Keywords—modelling, sea-clutter, bi-modal

I. INTRODUCTION

Producing simulated sea clutter requires accurate representation of a number of characteristics including the amplitude statistics, short-term temporal correlation (including that represented by Doppler spectra) and spatial and longer-term temporal variations. It must also represent the variation of range and azimuth over time as observed from a wide-area surveillance radar and model the effect of platform motion on the radar returns.

In the literature, the mean Doppler spectrum is often characterised by a single Gaussian component with an offset and spread [1]. This is in contrast to the asymmetrical spectra which are often observed due to the interplay of both fast and slow scattering mechanisms [2]. The slow scattering response is primarily associated with resonant Bragg scattering from wind induced capillary wave structures on the sea surface and exhibits Doppler shifts on the order of tens of Hertz (for X-band systems), consistent with the anticipated phase velocity of capillary waves. While the Bragg scattering response typically dominates for vertically polarised radar systems, studies of low grazing angle data have shown that horizontal polarised systems can be heavily affected by fast scattering mechanisms, so named for the higher Doppler frequencies they exhibit in comparison with Bragg scattering.

Many authors investigating sea-spike observed that there is a degree of polarisation independence in the non-Bragg component when looking at sea-clutter backscatter. Jessup, et al. [3] observed that as the grazing angle increased, the horizontal and vertically polarised returns became similar. This influenced the model presented by Walker [4] which uses a combination of three Gaussian components to describe the Doppler spectrum. This included two components for the non-Bragg scatterers to model the persistent polarisation independent return from the breaking waves (whitecaps) and the discrete short lived spikes.

At medium grazing angles, Rosenberg et al. [5] fitted the Ingara medium grazing angle data to the Walker model with good agreement. However further investigations [6] found that the Walker model did not totally describe the scattering and a modification was required. Consequently, a new two component Doppler spectrum model was introduced [7] using the Gaussian building blocks that both Walker [4] and Lamont-Smith [8] used. McDonald and Cerutti-Maori then improved the two-component fit by including the effective shape parameter for each Doppler bin in the model fitting [9]. In their model, the sea-clutter is described by the sum of two K-distributed variates which are associated with the Bragg and fast scattering components.

While these models may be useful to capture the mean of the Doppler spectrum, realistic simulation also requires changes in both range and time. Therefore, to achieve realistic simulation of sea-clutter and accurately determine the detection performance of a coherent radar, it is important to correctly model this evolution. There are only a few models designed to capture these characteristics including the work by Greco et al. [10], Watts [11]–[13] and more recently by McDonald and Cerutti-Maori [14]. The later model was demonstrated using parameters estimated from a vertically polarised dataset collected by Fraunhofer FHR’s multi-channel PAMIR radar with a good agreement between real and simulated data. The model has a stronger physical basis than the evolving Doppler spectrum model, but needs further investigation to determine its accuracy and robustness. This paper explores how well the model can represent the Ingara sea clutter over a range of swell angles and polarisations. It is directly contrasted with the evolving Doppler spectrum model which has previously been demonstrated to match the characteristics of the Ingara sea clutter with good accuracy [12], [13]. These two models are further described in Section II. Section III then describes the Ingara data with details of the estimated model parameters. An analysis of how well the model can represent the Ingara data set is then presented in Section IV.

II. MODEL SUMMARY

A. Evolving Doppler spectrum model

There has been extensive work to validate and extend the evolving spectrum model [11]. This includes assessment of the
CSIR dataset [15], the NetRAD dataset [16] and the Ingara dataset [12]. This model has been extended for multiple phase centres in [17] and for a multiple phase centre scanning radar in [18]. A bimodal extension to the model was also presented in [13] and is the most suitable version for the comparison here. It is described by a mixture model of two Gaussian-shaped power spectra designed to model the Bragg and fast components respectively. The model is given by

\[
G(f, x, s_1, s_2) = \frac{\alpha x}{\sqrt{2\pi s_1}} \exp\left[-\frac{(f - m_1(x))^2}{2s_1^2}\right] + \frac{(1-\alpha)x}{\sqrt{2\pi s_2}} \exp\left[-\frac{(f - m_2(x))^2}{2s_2^2}\right]
\]

where \(\alpha\) is a weighting factor between the two components, \(s_1\) and \(s_2\) are the spectrum widths and the centre frequency is related to the normalised mean speckle power, \(x_n = x/\langle x\rangle\) with

\[
m_1(x) = \begin{cases} A + Bx_n + r, & x_n \leq t_{b1} \\ A + Bt_{b1} + r, & x_n > t_{b1} \end{cases}
\]

\[
m_2(x) = A + Bx_n + r.
\]

where \(r\) is a zero mean Gaussian random variable with variance \(\sigma_r^2\). This model for the Doppler spectrum has the effect of broadening the spectrum if \(x_n\) exceeds the threshold, \(t_{b1}\). The spectrum width can also be assumed equal, \(s = s_1 = s_2\) and modelled as a random variable with mean, \(m_s\) and variance, \(\sigma_s^2\). The original work in [11] proposed a Gaussian distribution for the width, although a gamma distribution may be preferable to ensure \(s\) is positive [12].

In addition, the texture is correlated over time and range with a specified distribution. To achieve a K-distribution for the overall clutter signal, it can be modelled with a gamma distribution

\[
P(x) = \frac{b^\nu}{\Gamma(\nu)} x^{\nu-1} \exp\left[-bx\right], \quad \nu, b > 0
\]

where \(\nu\) and \(b = \nu/p_c\) are the shape and scale parameters respectively with the mean given by \(p_c\).

To simulate sea clutter using this model, one method is to first create a realisation of the Doppler spectrum, \(G(f)\) in (1) over range or time with a coherent processing interval of \(M\) pulses.

\[
Y_{evolved}(f) = \sqrt{G(f, x, s)} C(0, 1) + C(0, P_n)
\]

where \(P_n\) is the noise mean power in the frequency domain and \(C(0, p)\) are realisations of complex normal random variables with zero mean and variance \(p\). The time domain signal is then obtained by an inverse Fourier transform of (4).

**B. Uniformly distributed model**

The two component model for the mean Doppler spectrum [7], [9] is based on the idea that the sea clutter backscatter arises from two different scattering mechanisms (Bragg and fast scattering (FS)), with each mechanism giving rise to an inherent Gaussian spectral response with a total mean clutter spectrum given by:

\[
S_{total}(f) = x_B S_B(f) + x_{FS} S_{FS}(f)
\]

where \(S_B(f)\) and \(S_{FS}(f)\) represent the Bragg and fast spectra respectively with mean powers given by \(x_B\) and \(x_{FS}\). The Gaussian spectra are represented by

\[
S(f) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(f - \bar{f})^2}{2\sigma^2}\right]
\]

with mean \(\bar{f}\) and width \(\sigma\). For the dual-polarised data set studied in this paper, the model is slightly extended using the formulation in [7] where the spectral components have a common centre frequency and width.

The work by McDonald and Cerutti-Maori [9] proposed that the texture be represented by a gamma distribution with shape \(\gamma\). Each component is then described by a K-distribution with the overall clutter model having a K+K distribution. This model is referred to as the uniform model as the spectral and stochastic components are uniformly distributed both in azimuth and range.

**C. Spatially and temporally limited clutter (STLC) model**

The fast scattering is often associated with sea spikes and can be distinguished from Bragg scattering by a number of factors including the magnitude of the sea clutter, the Doppler characteristics and the persistence of scatterers over time [6].

The STLC model is an extension of the uniform model, except now the Bragg component, \(z_B\), is associated with scattering off the persistent waves with tilted wave-like features and variations across both range and Doppler. The fast scattering component, \(z_{FS}\), then models the response from the discrete spikes without range spreading or significant Doppler spread [14]. The complete STLC time domain signal of size \(N \times 1\) is then given by the coherent sum of the two components with additive Gaussian noise,

\[
Y_{STLC} = z_B + z_{FS} + C(0, p_c).
\]

The model can be used to simulate multi-channel data, but in this paper, we limit the scope to only a single channel. In this section, we now outline the model with extra details inferred from the original publication [14].

1) Bragg scattering component: To achieve the wave-like patterns observed in some Doppler spectra, the Bragg scattering component requires a model of the underlying wave pattern of the sea. An example is shown in the top plot of Fig. 1 as a function of range and azimuth. In this model, the azimuth angle, \(\theta\), is defined as the angular change between the radar position and scene centre over the collection time. The wave pattern is dependent on wavelength and both the range and azimuth extent of the data being characterised. It can greatly affect the result of the STLC Doppler spectrum. For this example, the wave pattern is defined over 75 m in range and an azimuth extent of 3.1° with a spacing of 41.4 m in range and 1.7° in azimuth.
The Bragg component is modelled as a complex Gaussian random variable of size $M \times 1$ with probability

$$P(z_B) = \frac{1}{(\pi x_B)^{M/2} |R_B|} \exp \left(-\frac{x_B^H (x_B R_B)^{-1} x_B}{2} \right)$$

(8)

where $x_B$ is the mean power of the Bragg component and $R_B$ is the covariance. The first step in determining the covariance is to consider each range bin separately and define the location of Bragg scattering events at the minima of each wave slice. These events then have an associated Bragg spectrum $S_B(f)$ located at sequential positions in Doppler and truncated to a width of $N_\sigma$ times the Bragg spectral width. This is shown for the first range bin in the second plot of Fig. 1 with the mapping between Doppler and azimuth angle shown by the arrows. The relationship between Doppler frequency and azimuth angle is given by $f = 2v_p \sin \theta/\lambda$, where $\lambda$ is the radar wavelength and $v_p$ is the platform velocity.

The covariance, $R_B$ is then the summation of $P$ events observed over azimuth in a single range cell,

$$R_B = \sum_{p=1}^P R_B(p)$$

(9)

where the covariance of each event is given by

$$R_B(p) = \sum_{n=1}^{N_{wave}} R_f(p, n) \otimes R_\theta(p, n)$$

(10)

where $\otimes$ is the Kronecker function, $N_{wave}$ is the number of points mapped between Doppler and azimuth (i.e. the arrows shown in Fig. 1) and the spatial component is equal to the azimuth antenna pattern, $R_\theta = A(f(p, n))$ for a single channel system. The temporal component is then defined by

$$R_f(p, n) = s(f(p, n))s^H(f(p, n))$$

(11)

with steering vectors given by

$$s(f) = [1, \exp(-j2\pi fT), \ldots, \exp(-j2\pi f(M-1)T)]$$

(12)

where $T$ is the pulse repetition interval. The individual Doppler frequencies are given by

$$f(n, p) = f_B(n) + f_\theta(p)$$

(13)

where $f_B(n)$ are the Doppler frequencies associated with the Bragg spectrum and $f_\theta(p)$ is the shift associated with the central azimuth location of each Bragg event. These are the minima shown in the bottom plot of Fig. 1.

The steps for simulating the Bragg component include:

- Create a realisation of the underlying wave over a defined range and azimuth extent.
- Determine the Doppler mapping for each event, $f(n, p)$.
- Generate correlated complex Gaussian realisations, $z_B$ to model the fast scattering events with mean power determined by $x_B$.
- Form the corresponding covariance matrix $R_B$.
- Generate correlated complex Gaussian realisations, $z_B$ to model the fast scattering events with mean power determined by $x_B$.

2) Fast scattering component: Similar to the uniform model, the fast scattering mechanism is modelled as a K-distribution, except its existence is contingent on a second random variable with a Bernoulli distribution. This random variable is referred to as the probability of occurrence, $P_{occ}(\theta)$. At each range bin, the received fast scatterer vector of size $M \times 1$ is given by the sum of realisations for each fast scattering event, $z_{FS}(\theta)$. These each have probability

$$P(z_{FS}(\theta)) = P_{occ}(\theta) \int_0^{\infty} \frac{1}{(\pi x_{FS})^{M/2} |R_{FS}(\theta)|} \exp \left(-\frac{x_{FS}^H (x_{FS} R_{FS}(\theta))^{-1} x_{FS}}{2} \right) P(x_{FS}) dx_{FS}$$

(14)

where $P(x_{FS})$ is modelled as a gamma distribution with shape parameter $\nu_{FS}$. The covariance matrix is defined for a single azimuth location as $R_{FS}(\theta) = (R_T \otimes R_\theta)$, with the temporal component defined by

$$R_T = \int f S_{FS}(f)s(f)s^H(f) df.$$
III. PARAMETER ESTIMATION

In August 2004 and July 2006, the DST Group’s Ingara X-band airborne radar collected fine resolution fully polarimetric data off the coast of Australia. The data was collected from an aircraft flying a circular track with a side-looking antenna pointing at a patch of sea, providing continuous data over 360° of azimuth and grazing angles between 15° and 45°. For each data set, the radar noise level in the absence of clutter was also recorded. The radar operated with a 200 MHz bandwidth centred at 10.1 GHz and a PRF of 578 Hz in the dual polarimetric mode.

The data analysed here comprises a time interval of 2 seconds and 31°–36° in grazing or 450 m in range. The wind and swell directions are aligned and the sea comes from a Douglas sea state 3-4 with a wind speed of 10.2 m/s and wave height of 1.21 m. Prior to any measurement of the Doppler domain, a coherent processing interval of N = 64 pulses is extracted from the data block and a -55 dB Doppler-Chernyshov window is applied. The Doppler spectrum has also been centred due to an unknown offset during the data collection and all model fits and simulations are based on the observed Doppler spectrum which includes broadening due the radar platform motion. To evaluate the two models, the Ingara parameters are estimated for both the horizontal transmit and receive (HH) and vertical transmit and receive (VV) polarisations and for three swell angles, θsw = 0°, 45° and 90°. Table I shows the key estimated parameters for each model.

Details on how to estimate the parameters for the evolving Doppler spectrum model are given in [12] and are not repeated here. The uniform model parameters are determined using a least squares model fit for both the two component Doppler model in (5) and an equivalent relationship for the spectrum variance. This technique for fitting the model was developed in [19] and does not require direct use of the effective shape parameters. An example model fit for the HH polarisation at θsw = 0° is shown in Fig. 2.

Over the 2 s data collection time, the STLC model has a wave pattern shown in the top part of Fig. 1. The occurrence of discrete spikes can be determined using the method described in [6], where the intensity data is thresholded at 5 standard deviations above the mean. Analysis of the discrete spikes distribution revealed that a negative exponential model would match better than the gamma distribution suggested in [14].

The swell spacing or wavelength in range is determined by fitting a model to the spatial correlation in the upswell direction. We can use a variation of the model presented in [20] where R(x) has a number of coefficients a, b1, b2 and a wavelength Rwave.

\[ R(x) = (1 - a) \exp(-b_1 x) + a \exp(-b_2 x) \cos \left( \frac{2\pi x}{R_{\text{wave}}} \right) \]  \hspace{1cm} (16)

For the data used here, the wavelength was estimated as \( R_{\text{wave}} = 29.3 \text{ m} \). Finally, the number of points used to map between the Bragg events and the Bragg spectrum, \( N_{\text{wave}} = 30 \) and the truncated width of the Bragg component is \( N_{\theta} = 3 \) times the standard deviation of the Bragg component estimated by the uniform fit.

IV. MODEL ANALYSIS

This section now considers realisations of both models using a CPI of 64 pulses and 600 range bins. To assess the model over the chosen swell directions and polarisations, we will first simulate data for each case and then estimate the spatial decorrelation length, the K-distribution shape using the \( z_{\log z} \) estimator and measure the root mean square (RMS) error of the simulated shape in the endo-clutter region of the frequency domain. The Doppler spectrum plots are shown in Figs. 3-5 for swell directions, \( \theta_{\text{sw}} = 0^\circ, 45^\circ \) and \( 90^\circ \) with the measured results in Table II.

For each result, the evolved Doppler spectrum model has produced a good representation of the data, although it does not reproduce the tilted range-Doppler pattern seen in the data.
The measured shape and spatial decorrelation lengths are also quite close for each result. For the STLC results, the upswell results, $\theta_{sw} = 0^\circ$, have produced strong Bragg components at each wave location with a broad fast component. These results also have an extremely low shape and spatial decorrelation.

For the $\theta_{sw} = 45^\circ$ results, the STLC model has produced a tilted wave structure in the spectrum. This has resulted in a good match with the shape estimate for the HH polarisation. To determine the exact wave structure which matches the estimated shape in the data is extremely difficult, as the sea clutter will likely contain a more complicated wave structure than can be represented by the STLC model. The final result with $\theta_{sw} = 90^\circ$ has artificial edges to the spectrum, which are more pronounced for VV polarisation.

The final results in Fig. 6 show the K-distribution shape estimates over frequency. For the evolved Doppler model, the majority of estimates are extremely close. For the STLC model, the only results which match the data are for $\theta_{sw} = 45^\circ$. For the other results, there are some frequency regions which match, but not across the entire spectrum. Although not shown, the clutter to noise ratio across frequency is also important to accurately model the coherent properties of the data. For the evolved spectrum model and the STLC model with $\theta_{sw} = 45^\circ$, there is a good match with the data across the spectrum. However, this is not the case for the other STLC model results, where a significant mismatch is observed.

Fig. 3. Doppler spectrum comparison for swell angle $\theta_{sw} = 0^\circ$.

Fig. 4. Doppler spectrum comparison for swell angle $\theta_{sw} = 45^\circ$.

Fig. 5. Doppler spectrum comparison for swell angle $\theta_{sw} = 90^\circ$. 
TABLE II. REAL AND SIMULATED PARAMETER COMPARISON.

<table>
<thead>
<tr>
<th>Polarisation</th>
<th>HH</th>
<th>VV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swell angle (deg)</td>
<td>0°</td>
<td>45°</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cutter to noise ratio (dB)</td>
<td>15.4</td>
<td>12.4</td>
</tr>
<tr>
<td>K-dist. shape</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Spat. decorr. length (m)</td>
<td>1.2</td>
<td>1.9</td>
</tr>
<tr>
<td>Evolved Doppler model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K-dist. shape</td>
<td>3.9</td>
<td>5.2</td>
</tr>
<tr>
<td>Spat. decorr. length (m)</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>K-dist. shape (freq.) RMS error</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>STLC model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K-dist. shape</td>
<td>0.08</td>
<td>4.1</td>
</tr>
<tr>
<td>Spat. decorr. length (m)</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>K-dist. shape (freq.) RMS error</td>
<td>1.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Fig. 6. K-distribution shape variation over frequency.

V. CONCLUSION

This paper has examined two models for representing sea-clutter with a bi-modal spectrum. Through the analysis in this paper, the evolved Doppler spectrum has consistently produced good results. The STLC model produced a realistic looking tilted wave spectrum for $\theta_{sw} = 45^\circ$, with a good match for the shape across frequency. However, the upswell spectrum does not appear realistic and has poor estimates across frequency. Also, for the STLC results, the overall shape was wrong and the spatial correlation is not being modelled correctly, indicating that we are not modelling the wave pattern of the data correctly. Future work will examine the performance of coherent detection schemes against both simulated and real data. We also plan to examine how the STLC model can be modified to maintain the distinctive tilted wave spectrum and incorporate some of the features from the evolved Doppler spectrum model.

REFERENCES