Joint optimization of transmit waveform and mismatched filter with expanded mainlobe for delay-Doppler sidelobes suppression

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***Abstract*—Low range sidelobe level is a main pursuit in radar waveform optimization, especially at the target searching mode. In this paper, we propose an approach to jointly optimize a transmit waveform and its mismatched filter to suppress range sidelobes even with slight Doppler mismatch under constant modulus constraint. Specifically, we propose a minimization criterion to suppress delay-Doppler sidelobes and control signal-to-noise ratio loss. Since the design criterion is a non-convex optimization problem, we exploit a double least-*p*th minimax algorithm to solve it. Numerical results indicate that the approach can efficiently suppress delay-Doppler sidelobes over different specifications of the Doppler mismatch.**

Keywords—Radar transmit waveform; mismatched filter; simultaneous optimization; delay-Doppler sidelobes

I. INTRODUCTION

As one major problem in radar signal processing, waveform design has attracted much attention in the past decades [1-6]. For large time-bandwidth waveforms, pulse compression ia necessary, which inevitably produces range sidelobes. A high-range sidelobe level may trigger false alarms or cause high-power target returns submerging low-power target returns in range bins nearby, achieving desired correlation property of waveforms is a fundamental requirement. Generally, peak sidelobe level (PSL) and integrated sidelobe level (ISL), which serve as two evaluation metrics for correlation property of waveforms, are usually considered and suppressed to achieve desired correlation properties. In this paper, we consider the PSL as the main evaluation metric of sidelobe levels.

Phase coded waveform (PCW) is a kind of spread spectrum signal with good anti-interference, low probability of interception and high measurement accuracy. At present, PCW design with low sidelobes has been extensively studied [1-6] including polyphase codes and continuous phase codes. With the development of science and technology, advances in radar hardware make it possible to use direct digital synthesizer (DDS) to generate continuous phase codes. To further suppress sidelobes, there may exist two ways. One is increasing the number of symbols inside a PCW. However, for a pulse with fixed time duration, increasing the number of symbols inside a waveform may reduce the width of mainlobe. Such kind of mainlobe distortion is not desired in some particular applications [6]. The other way for reducing PSLs is designing a mismatched filter with extra control in signal-to-noise ratio (SNR) loss [7]. However, both [6] and [7] aforementioned did not take Doppler mismatch into consideration, and the sidelobe levels would increase rapidly even with a small Doppler mismatch. Instead, to handle the sensitivity of PCW in Doppler mismatch, [8] considers designing a Doppler robust mismatched filter with given transmit waveform.

Different from the works in [6-8] which only focus on suppressing PSLs or dealing with Doppler sensitivity, an approach of simultaneous optimization of transmit waveform and mismatched filer with robustness to Doppler mismatch is considered in this paper. We put great emphasis on two sides about the proposed approach. The first one is expanding mainlobe: We can understand it from the following three major aspects: 1) For fixed duration time of transmit waveform (FDTTW), the mainlobe width of a PCW after pulse compression becomes smaller gradually as the number of codes increases; 2) For FDTTW, we demand that the range resolution should agree with the prescribed one when increasing the number of codes for lower sidelobe levels. As such, we attempt to expand mainlobe width as the number of codes increases; 3) It should be noted that the bandwidth of the devised PCW with expanded mainlobe (PCWEM) does not approximately equal that of the subpulse. In addition, this kind of waveform does not violate the relationship between bandwidth and range resolution. The bandwidth can be controlled by prescribing the mainlobe width, which is verified by numerical results below. Since such kind of waveforms can insert more codes within a waveform, we can obtain more degrees of freedom (DOF) available. Hence, it is reasonable to yield lower sidelobe levels. The second one is the Doppler range for suppressing delay-Doppler sidelobes. It is known that in the searching mode, we often have certain knowledge about the maximum radial velocities of targets of interest, and in the tracking mode, some samples can be used to estimate and predict the radial velocity information of targets of interest. In either case, a range of Doppler frequencies can be roughly obtained. Therefore, keeping Doppler robustness over the interested range is a more efficient way.

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Combining the two sides mentioned above, we propose a minimization criterion to simultaneously suppress delay-Doppler sidelobes, match desired mainlobe, control SNR loss, and keep Doppler robustness over an interested Doppler range. Based on the least-*p*th minimax algorithm [9], we propose a double least-*p*th minimax (DLPM) algorithm with the Limited-Memory Broyden-Fletcher-Goldfarb and Shannon (L-BFGS) [10] algorithm as its subalgorithm to address it.

II. Problem Formulation

Consider a radar system with a PCW $\mathbf{s} \in \mathbb{C}^{N_s}$ and a mismatched filter $\mathbf{h} \in \mathbb{C}^{N_h}$ ($N_h \geq N_s$). The PCW $\mathbf{s}$ after Doppler modulation can be expressed by

$$\overline{\mathbf{s}} = \mathrm{diag}\left[\boldsymbol{\kappa}\left(f_d\right)\right]\mathbf{s} \tag{1}$$

where $\boldsymbol{\kappa} = \left[1, e^{j2\pi f_d}, e^{j2\pi 2 f_d}, \cdots, e^{j2\pi(N_s-1)f_d}\right]^T$ is the Doppler steering vector, $(\cdot)^T$ is the transpose operation, f_d denotes the normalized Doppler frequency and $\mathrm{diag}(\cdot)$ with a vector entry denotes a diagonal matrix with the entry vector as diagonal elements. The *k*th component of PCW $\mathbf{s}$ is

$$\mathbf{s}(k) = \begin{cases} e^{j\boldsymbol{\varphi}(k)}, & \text{for } k \in [1,\ N_s] \\ 0, & \text{others} \end{cases} \tag{2}$$

where $\boldsymbol{\varphi}$ is the phase vector of the PCW $\mathbf{s}$, i.e., $\mathbf{s} = \exp(j\boldsymbol{\varphi})$. The Doppler modulated PCW $\overline{\mathbf{s}}$ after mismatched filtering $\mathbf{h}$ at delay τ can be written as

$$\omega(\tau, f_d) = \sum_{k=-\infty}^{\infty} \overline{\mathbf{s}}(k)\mathbf{h}^*(k-\tau) \tag{3}$$

where $(\cdot)^*$ denotes the conjugate operation. The *k*th component of mismatched filter $\mathbf{h}$ is

$$\mathbf{h}(k) = \begin{cases} \mathbf{a}(k)e^{j\boldsymbol{\theta}(k)}, & \text{for } k \in [1,\ N_h] \\ 0, & \text{others} \end{cases} \tag{4}$$

where $\mathbf{a}$ and $\boldsymbol{\theta}$ are the magnitude and phase vectors of the mismatched filter $\mathbf{h}$, i.e., $\mathbf{h} = \mathbf{a} \odot \exp(j\boldsymbol{\theta})$. In this paper, for simultaneous optimization of PCW and mismatched filter, three major properties are considered:

(1) Mainlobe width control: Assume that the radar transmit end is capable of transmitting N_s codes under FDTTW. With these codes, we need to match a prescribed range resolution. For general PCW, the range resolution can be evaluated by

$$\Delta\mathrm{R} = \frac{\mathrm{c}T}{2N_s} \tag{5}$$

where c is light speed and T denotes the transmit waveform duration time. The prescribed range resolution is denoted by

$$\Delta\mathrm{R}_d = \frac{\mathrm{c}T}{2N_1} \tag{6}$$

where N_1 ($N_s \geq N_1$) is the number of transmitting codes. Hence, the increased multiple of range resolution is

$$g = \frac{\Delta\mathrm{R}_d - \Delta\mathrm{R}}{\Delta\mathrm{R}} = \frac{N_s - N_1}{N_1} \tag{7}$$

Under FDTTW, we resort to expanding mainlobe width when increasing the number of codes inside a PCW. To control mainlobe shape effectively, let $\mathbf{p} \in \mathbb{C}^{2M+1}$ be a desired mainlobe, which could be the mainlobe of sinc function or other functions, and M represents the width of desired mainlobe. In practice, M can be calculated by

$$M = \mathrm{ceil}(\gamma \cdot g) \tag{8}$$

where $\mathrm{ceil}(\cdot)$ means rounding to the nearest integer toward positive infinity, γ is an empirical coefficient (usually $\gamma = 0.65$).

The *m*th component of matching error vector between the devised mainlobe and desired mainlobe can be expressed as

$$\mathbf{e}(m) = \omega(m - M - 1, 0) - \mathbf{p}(m) \tag{9}$$

where $m = 1, 2, \cdots, 2M+1$. For all components of matching error vector, the maximum one can be denoted by $\|\mathbf{e}\|_\infty, \forall m$, where $\|\cdot\|_\infty$ denotes the infinity norm. The mainlobe shape can be controlled by

$$\min_{\boldsymbol{\varphi}, \mathbf{a}, \boldsymbol{\theta}} \ \|\mathbf{e}\|_\infty. \tag{10}$$

(2) SNR loss control: Mismatched filter design usually encounters SNR loss. To avoid unacceptable SNR loss, we propose a simple but efficient approach to control SNR loss. Under independent and identically distributed white Gaussian noise, the SNR loss [11] of zero Doppler channel is

$$\mathrm{SNR}_{\mathrm{loss}} = 10\log_{10}\left(N_s \mathbf{h}^H \mathbf{h} \big/ \left(\omega^*(0,0)\,\omega(0,0)\right)\right) \tag{11}$$

where $(\cdot)^H$ is the conjugate transpose operation. To avoid the peak value loss, assume $\omega(0,0) = N_s$ ($\mathbf{s}^H\mathbf{s} = N_s$), which implies that the peak value of PCW after mismatched filtering is equal to that after matched filtering. Hence, the SNR loss is simplified into

$$\mathrm{SNR}_{\mathrm{loss}} = 10\log_{10}\left(\mathbf{h}^H\mathbf{h}/N_s\right). \tag{12}$$

Based on (12), the SNR loss decreases as the $\mathbf{h}^H\mathbf{h}$ reduces. Notice that the assumption $\omega(0,0) = N_s$ is a nonconvex constraint, which is hard to handle in optimization. Hence, we propose to control the SNR loss by simultaneously minimizing $\left\|\mathbf{h}^H\mathbf{h} - N_s\right\|_\infty$ and $\left\|\omega(0,0) - N_s\right\|_\infty$. Besides, substituting $m = M+1$ into (9), we can obtain $\mathbf{e}(M+1) = \omega(0,0) - N_s$ (i.e., $\|\mathbf{e}\|_\infty$ covers $\left\|\omega(0,0) - N_s\right\|_\infty$). Therefore, besides (10), the SNR loss is also controlled by

$$\min_{\boldsymbol{\varphi}, \mathbf{a}, \boldsymbol{\theta}} \ \left\|\mathbf{h}^H\mathbf{h} - N_s\right\|_\infty. \tag{13}$$

(3) Delay-Doppler sidelobes suppression: Either in searching mode or tracking mode, the range of Doppler frequencies of interest can be roughly obtained in priori. In practice, keeping Doppler robustness over the interested Doppler range is a more efficient way. To simplify the notation, assume that Ω is the given range of normalized Doppler frequency, which is divided into N_d Doppler frequencies. In addition, define

$\Gamma=\left[-\left(N_{\mathrm{s}}+N_{\mathrm{h}}\right)/2+1,\cdots,-M-1,M+1,\cdots,\left(N_{\mathrm{s}}+N_{\mathrm{h}}\right)/2-1\right]$. Stack all the delay-Doppler sidelobes into a column vector $\boldsymbol{\omega}_{\text{side}}$ ($\forall f_{\mathrm{d}}\in\Omega, \forall\tau\in\Gamma$). To achieve low PSL, we propose a min-max optimization criterion:

$$\min_{\boldsymbol{\varphi},\mathbf{a},\boldsymbol{\theta}} \max_{f_{\mathrm{d}}\in\Omega,\ \tau\in\Gamma} \left|\boldsymbol{\omega}_{\text{side}}\right|. \quad (14)$$

The objective function in (14) can be written as $\left\|\boldsymbol{\omega}_{\text{side}}\right\|_{\infty}$.

Combining (10), (13) and (14), the simultaneous optimization of PCW and mismatched filter can be cast as

$$\min_{\boldsymbol{\varphi},\mathbf{a},\boldsymbol{\theta}} \left\|\left\|\boldsymbol{\omega}_{\text{side}}\right\|_{\infty}, \alpha\cdot\left\|\mathbf{e}\right\|_{\infty}, \beta\cdot\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{\infty}\right\|_{\infty} \quad (15)$$

where α and β are pre-specified positive real numbers for the tradeoff among the delay-Doppler sidelobes suppression, SNR loss and matching mainlobe performance.

III. Optimization Method

Since (15) is a nonconvex and unsmooth optimization problem, we propose a double least-*p*th minimax algorithm to solve it. Specifically, the minimization $\left\|\boldsymbol{\omega}_{\text{side}}\right\|_{\infty}$, $\left\|\mathbf{e}\right\|_{\infty}$ and $\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{\infty}$ can be achieved by minimizing $\left\|\boldsymbol{\omega}_{\text{side}}\right\|_{p}$, $\left\|\mathbf{e}\right\|_{p}$ and $\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{p}$ with increasing value of p [9, 12], where $\left\|\cdot\right\|_{p}$ denotes the L_p norm. Define

$$f(\mathbf{x})=\left\|\left\|\boldsymbol{\omega}_{\text{side}}\right\|_{p}, \alpha\cdot\left\|\mathbf{e}\right\|_{p}, \beta\cdot\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{p}\right\|_{p} \quad (16)$$

where $\mathbf{x}$ is a column vector consisting of $\boldsymbol{\varphi}$, $\mathbf{a}$ and $\boldsymbol{\theta}$ in turn. Minimizing $f(\mathbf{x})$ can be solved by the proposed DLPM algorithm, which are given in Table I. Parameter μ in step 1 denotes that p increases by μ times between two successive iterations, which should be an integer obviously and not too large in order to avoid numerical ill-conditioning. The μ with the value of 2 gives good results [9]. f_0 is an arbitrary value greater than the possible maximum value of $f(\mathbf{x})$ and we set $f_0=1000$ here. The minimization problem in step 2 can be solved by the L-BFGS algorithm [10].

TABLE I. The Procedure of DLPM Algorithm

Step 1: Input $\mathbf{x}_0$ and ε. Set $w=1$, $p=2$, $\mu=2$, and $f_0=1000$.
Step 2: Using $\mathbf{x}_{w-1}$ as an initial point, minimize $f(\mathbf{x})$ with respect to $\mathbf{x}$, to obtain $\mathbf{x}_w$. Set $f_w=f(\mathbf{x}_w)$.
Step 3: If $\left|f_{w-1}-f_w\right|<\varepsilon$, then output $\mathbf{x}_w$, and stop. Otherwise, set $p=\mu p$, $w=w+1$, and go to step 2.

In the L-BFGS algorithm, the computational cost of each L-BFGS iteration is dominated by the evaluation of f and ∇f. For each iteration of p, the computational complexities (complex multiplications) f and ∇f of (16) are $O\left(N_{\mathrm{d}}\left(N_{\mathrm{s}}+N_{\mathrm{h}}\right)\left(p+\log_2\left(N_{\mathrm{s}}+N_{\mathrm{h}}\right)\right)+N_{\mathrm{h}}\right)$ and $O\left(N_{\mathrm{d}}\left(N_{\mathrm{s}}+N_{\mathrm{h}}\right)\right)$, respectively.

IV. Simulation Results

Throughout the simulations, unless otherwise explicitly stated, we consider a PCW with $N_{\mathrm{s}}=256$ and a mismatched filter with $N_{\mathrm{h}}=512$. The duration time of transmit waveform is 64μs. The sample rate is 12MHz. Without loss of generality, assume that the Doppler range of interest Ω is $\left[-f_{\text{dmax}}, f_{\text{dmax}}\right]$, where f_{dmax} is the maximum normalized Doppler frequency. The pre-mentioned values of distinct weighting coefficients in (15) will be given in the corresponding simulation. There are two termination conditions of the DLPM algorithm: one is 5000 to be reached for the maximum iteration number and the other is $\left\|\Delta f\right\|_F$ less than 10^{-4}, where $\left\|\cdot\right\|_F$ denotes the Frobenius norm.

To match a prescribed mainlobe width, we firstly design a general PCW according to (17) with $N_1=64$ solved by the same algorithm as (15)

$$\min_{\boldsymbol{\varphi}_1} \max_{r=1,2,\cdots,N_1-1} \left|\mathbf{s}_1^{\mathrm{H}}\mathbf{J}_r\mathbf{s}_1/N_1\right| \quad (17)$$

where $\boldsymbol{\varphi}_1$ is the phase vector of PCW $\mathbf{s}_1$, i.e., $\mathbf{s}_1=\exp\left(\mathrm{j}\boldsymbol{\varphi}_1\right)$, $\mathbf{J}_r$ is the shift matrix [6]. The mainlobe of PCW $\mathbf{s}_1$ after pulse compression will be considered as the prescribed mainlobe (i.e., desired mainlobe $\mathbf{p}$). For FDTTW, based on (8), we can obtain M=2. Now we utilize the PSL of Doppler modulated PCW after mismatched filtering within Ω as the performance metric, i.e.,

$$\mathrm{PSL}_{\Omega}=\max_{f_{\mathrm{d}}\in\Omega,\ \tau\in\Gamma} \left|\omega(\tau,f_{\mathrm{d}})/\omega(0,0)\right|. \quad (18)$$

Firstly, we present several instances by considering different α and β in Table II to validate the effectiveness of the proposed method to control SNR loss. Note that since $\left\|\mathbf{e}\right\|_{\infty}$ covers $\left\|\omega(0,0)-N_{\mathrm{s}}\right\|_{\infty}$, α can be used to adjust $\left\|\omega(0,0)-N_{\mathrm{s}}\right\|_{\infty}$. We can see that the SNR loss decreases by simultaneously minimizing $\left\|\omega(0,0)-N_{\mathrm{s}}\right\|_{\infty}$ and $\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{\infty}$.

Next, we give an example ($\alpha=1.68$ and $\beta=0.42$) to verify the effectiveness of the proposed method, where the Doppler range Ω is $\left[-1/2N_{\mathrm{s}},\ 1/2N_{\mathrm{s}}\right]$ with $1/4N_{\mathrm{s}}$ Doppler interval. The impacts of different Doppler range limits and Doppler intervals will be studied later.

TABLE II. Optimization Results of Different α and β

α	β	$\left\|\omega(0,0)-N_{\mathrm{s}}\right\|_{\infty}$	$\left\|\mathbf{h}^{\mathrm{H}}\mathbf{h}-N_{\mathrm{s}}\right\|_{\infty}$	SNR loss (dB)
1.48	0.22	30.4196	168.1691	1.2177
1.68	0.42	8.3951	34.2785	0.8354
1.88	0.62	7.9336	25.3563	0.6836
2.08	0.82	6.6291	17.0463	0.5078

Fig. 1 shows that, similar to [6], the mainlobe of the proposed method nearly coincides with the desired mainlobe. Meanwhile, for comparison purpose, we present the mainlobe of a random PCW with code length $N_{\mathrm{s}}=256$, whose phases are generated according to a uniform distribution over [0,2π]. It can be found that the random PCW has a smaller mainlobe width. Our waveform does not violate the relationship between bandwidth and range resolution. Although our waveforms have narrower code width, a combination of those codes generates a nonuniform frequency spectrum like general PCW $\mathbf{s}_1$. In order to verify this fact, the power spectrum of the devised PCW $\mathbf{s}$ after mismatched filtering $\mathbf{h}$ is shown in Fig. 2 together with that of general PCW $\mathbf{s}_1$. Fig. 2 indicates that this kind of PCW occupies only a limited frequency band. That is the key to keep the range resolution.

The delay-Doppler response of the devised PCW $\mathbf{s}$ after mismatched filtering $\mathbf{h}$ is shown in Fig. 3, and the corresponding PSLs of different Doppler frequencies are depicted in Fig. 4 together with that of the method in [6], which is defined as

$$\mathrm{PSL}\left(f_{\mathrm{d}}\right)=\max_{\tau\in\Gamma}\left|\omega\left(\tau,f_{\mathrm{d}}\right)/\omega\left(0,0\right)\right|. \tag{19}$$

Inspection of the curves highlights that, in contrast to the method in [6], the delay-Doppler sidelobes within Ω exhibit a smooth appearance (PSL_{Ω}=-24.6994dB and SNR loss= 0.8354dB). As expected, outside the Ω, the delay-Doppler sidelobes increase significantly over the Doppler shifts. But it has little influence on subsequent processing due to the fact that it has exceeded the maximum radial velocities of targets of interest.

During waveform optimization process, within Ω, we need to select a group of Doppler frequencies for delay-Doppler sidelobes suppression. It is vital to select proper Doppler interval because if the interval is too small, more elements should be suppressed, which results in considerable complexity, and if the interval is too wide, delay-Doppler sidelobes at intermediate Doppler frequencies may increase. Based on (15), we suppress delay-Doppler sidelobes over the Doppler range $\left[-2/N_{\mathrm{s}},\ 2/N_{\mathrm{s}}\right]$ with four different Doppler interval, i.e., $1/N_{\mathrm{s}}$, $1/2N_{\mathrm{s}}$, $1/4N_{\mathrm{s}}$ and $1/8N_{\mathrm{s}}$. We use PSLs and ISLs of different Doppler frequencies along range bins as the performance metrics and the ISL for different Doppler frequencies is defined as

$$\mathrm{ISL}\left(f_{\mathrm{d}}\right)=\sum_{\tau\in\Gamma}\left|\omega\left(\tau,f_{\mathrm{d}}\right)/\omega\left(0,0\right)\right|^{2}/N_{0} \tag{20}$$

where N_0 is the number of all range sidelobe bins ($\forall\tau\in\Gamma$).

Under nearly same mainlobe width and SNR loss (0.89 dB), for different Doppler sampling intervals, the PSLs and ISLs of different Doppler frequencies together with no Doppler consideration are depicted in Figs. 5 and 6, respectively. Firstly, compared Fig. 4 with Fig. 5, the PSL (zero Doppler channel) of proposed method with no Doppler consideration is much lower (about 10.68dB) than that of the method in [6] (red line), but the performance improvement is at the cost of SNR loss (0.89 dB). Secondly, from Figs. 5 and 6, we can see that when the interval is $1/N_{\mathrm{s}}$ or $1/2N_{\mathrm{s}}$, the sidelobe levels at selected Doppler frequencies can be suppressed, but for intermediate Doppler frequencies, the sidelobe levels may increase rapidly. The final PSLs of interval $1/4N_{\mathrm{s}}$ nearly equal that of interval $1/8N_{\mathrm{s}}$, but the number of elements for suppression is only a half. Therefore, we select Doppler sampling interval $1/4N_{\mathrm{s}}$ in the following simulations.

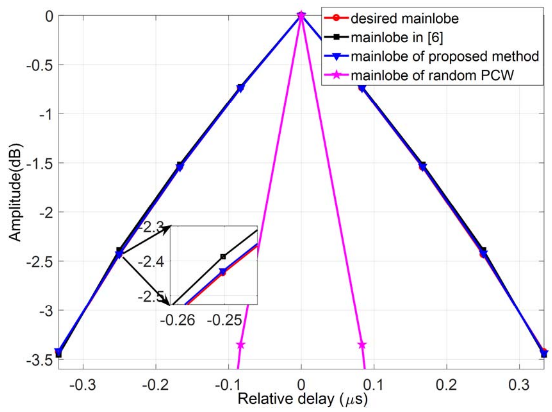

Fig. 1 Comparison of mainlobe amplitudes.

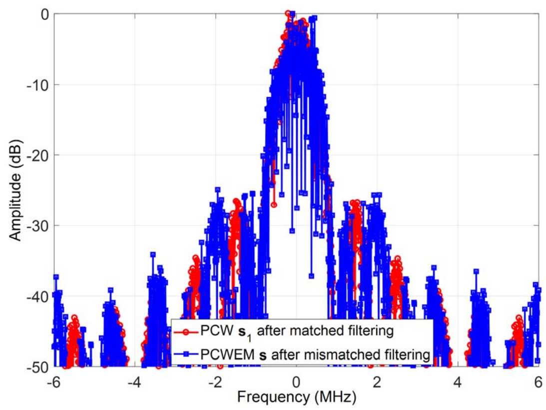

Fig. 2 Comparison of power spectrums.

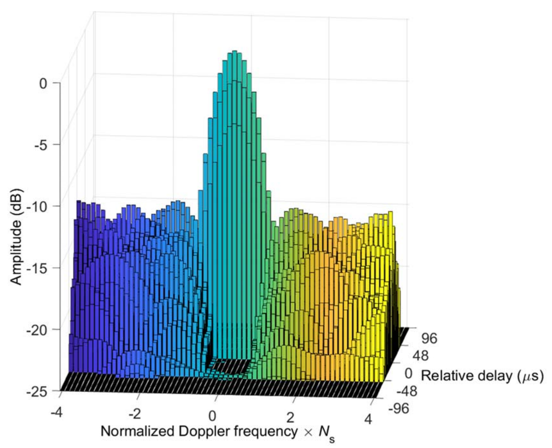

Fig. 3 Delay-Doppler response over Doppler frequencies $\left[-4/N_{\mathrm{s}},\ 4/N_{\mathrm{s}}\right]$.

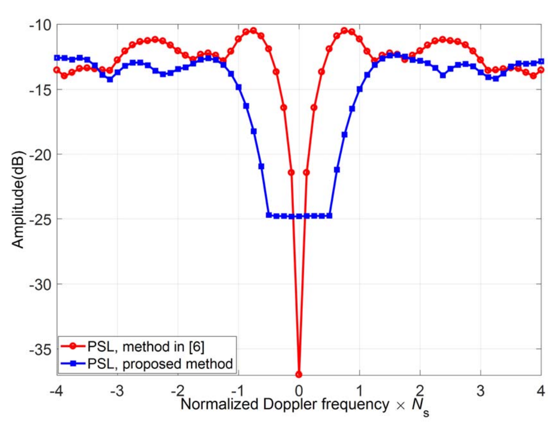

Fig. 4 Doppler robustness comparison of two methods.

Under nearly same mainlobe width and SNR loss (0.89 dB), for different Doppler range limits, the PSLs and ISLs of different Doppler frequencies are depicted in Figs. 7 and 8, respectively. Firstly, compared Fig. 4 with Fig. 7, the sidelobe levels for $f_{\text{dmax}} = 1/2N_s$ of Fig. 7 (red line, PSL_Ω=-25.1459dB and SNR loss=0.89dB) are lower than that of Fig. 4 (blue line, PSL_Ω=-24.6994dB and SNR loss=0.8354dB), which is at the cost of a larger SNR loss. Secondly, from Figs. 7 and 8, we find that a smaller the Doppler range means a lower PSL and ISL. Therefore, in practical applications, to achieve low PSL over a certain Doppler range, we can fully utilize priori knowledge of targets of interest to narrow the Doppler range. Besides, a range compression bank [13] can be used to compensate different Doppler shifts ensuring the Doppler range within a small range.

Given the prescribed range resolution (i.e., fixed N_1), the impacts of different M on PSL within Ω are studied. Since the M is related to the code length N_s according to (8), we provide the PSLs of different N_s (i.e., different M) in Table III. Table III indicates that, under nearly same mainlobe width and SNR loss (0.89dB), a larger M means a lower PSL and vice versa. This is due to the fact that, for FDTTW, increasing M is tantamount to increasing the code length of the devised PCW, i.e., more DOF. Hence, in practice, we can adjust M to obtain required results.

TABLE III OPTIMIZATION RESULTS OF DIFFERENT M

N_s	M	PSL_Ω (dB)
128	1	-19.3689
256	2	-25.1459
384	4	-27.0541

V. CONCLUSION

In this paper, we consider an approach of simultaneously optimizing PCW and mismatched filter to suppress delay-Doppler sidelobes. For FDTTW, this approach brings more DOF to suppress delay-Doppler sidelobes while keeping the same range resolution with the prescribed one. Numerical results validate that the proposed method not only can keep the range resolution with the prescribed one, but also can obtain good robustness over a specific Doppler shift. Moreover, the impacts of different Doppler range limits and Doppler sampling intervals on final sidelobes are studied. This research concerns with Doppler effects will promote the practical applications of PCWs in the field of radar.

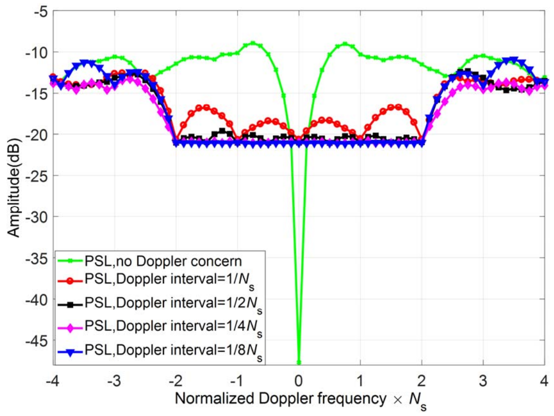

Fig. 5 PSL over range versus Doppler for different Doppler intervals.

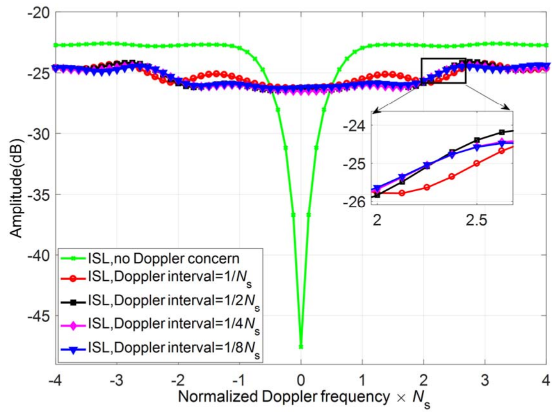

Fig. 6 ISL over range versus Doppler for different Doppler intervals

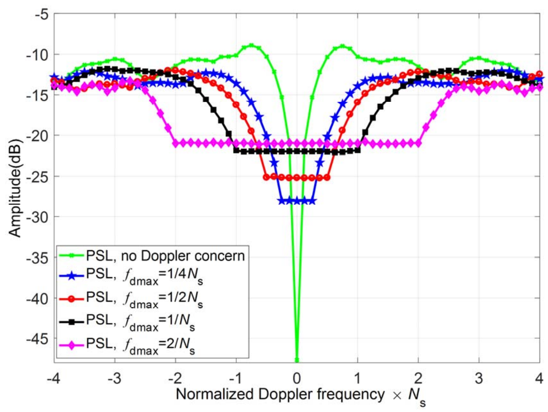

Fig. 7 PSL over range versus Doppler for different Doppler range limits.

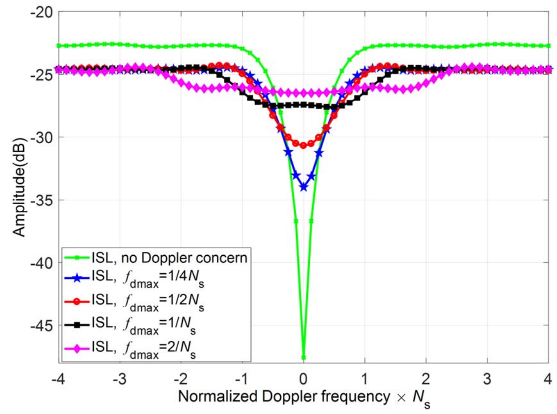

Fig. 8 ISL over range versus Doppler for different Doppler range limits

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