Colocated MIMO radar waveform design against repeat radar jammers

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***Abstract*—The digital radio frequency memory (DRFM) repeat jammer, which replicas the transmit signal and transmits back to radar receiver to create deceptive targets, is a cheap and quite destructive electronic attack. This paper concerns waveform design at the background of colocated multiple-input multiple-output (MIMO) radar to combat with the repeat jammers. Specifically, since the amplifiers of transmit circuits of repeat jammers often operate in the saturation mode, the repeat jamming signals should be constant modulus. However, for colocated MIMO radar, the synthetic angular waveform in space often is nonconstant modulus, which may have better performance in such environment. Under the assumption that hostile jammers operate in the saturation mode, we formulate a min-max design criterion that minimizes cross correlation between an angular waveform and a constant modulus waveform with the same phase terms. Numerical results validate the effectiveness of the proposed method.**

Keywords—DRFM; repeat jammer; collocated MIMO radar; waveform design; sidelobe suppresion

I. INTRODUCTION

With the rapid development of high speed signal processing hardware, the electronic attack (EA) of electronic warfare has achieved huge advances over the last several decades, which mainly transmits jamming signals to weaken and deteriorate the performance of radar system [1-3]. Digital radio frequency memory (DRFM) repeat jammer, which can replica and retransmit the intercepted signal, is a cheap and quite destructive EA [4-9]. In order to protect the true targets, DRFM repeat jammers resort to producing false targets in different range bins by properly delaying modulation and retransmitting the intercepted radar signals.

In recent years, some electronic protections (EP) have been proposed to weaken or eliminate the DRFM repeat jammer [5-9]. Pulse diversity is a common strategy to counter range repeat jammer and has been attracting much attention [6-9]. In [6], received signal is first matched filtered with the pulse used by the jammer to recognize false targets and remove them. Then, another matched filtering with regard to the current pulse discloses the true targets. Nevertheless, when the jamming signals have similar time delays to that of true targets, the real targets may mistakenly regard as false targets and be eliminated. In [7], the orthogonal properties of transmit waveforms are exploited to distinguish true targets and decrease the impacts of repeat jammers. However, the time of repeat jammer lag behind the radar is exactly know and only half the pulses in the block follow the orthogonal structure. In [8], a joint time-frequency pulse diversity canceller was proposed based on an orthogonal frequency division multiplexing under high signal-to-jammer ratio. In [9], robustness against jammer is obtained by designing different phase coded pulses with weighted auto/cross correlation in a coherent processing interval. But, during design process, the delay time of repeat jammer is constant or a random variable within an exactly known range. From the analysis aforementioned, most of existing works assume that the delay time of repeat jammer behind the radar lags at least one pulse repetition interval and is exactly known. In practice, for radar system, it is difficult to exactly obtain or know the delay time or delay time range of repeat jammer, which may even be within the duration time of current pulse.

In this paper, we outline a novel method based on colocated multiple-input multiple-output (MIMO) radar system whose distance between adjacent antennas is closely spaced [10, 11]. In general, for colocated MIMO radar, the waveforms illuminated into space are synthetized, which is called angular waveform subsequently. Note that the angular waveform often is nonconstant modulus. However, to maximize the transmit power, the amplifiers of transmit circuits of jammers often operate in the saturation mode. Hence, the repeat jamming signals should be constant modulus. To clearly and easily recognize true targets, on one hand, the transmit waveform need to have low auto/cross correlation sidelobes of angular waveforms and on the other hand, the cross correlation sidelobes between the angular waveforms and repeat jamming signals should be suppressed to weaken the ill-effects of repeat jamming signals [9]. Therefore, a min-max design criterion is formulated by matching desired beampattern, suppressing the auto peak correlation level (APSL), peak cross correlation level (PCCL) of angular waveforms and the jamming peak cross correlation sidelobe level (JPCCL) between the angular waveforms and repeat jamming signals. Simulation results verify the effectiveness and benefits of the proposed method.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A. SignalModel

Consider a colocated MIMO radar system equipped with N_t transmit antennas and N_r receive antennas. Let $\mathbf{s}_i \in \mathbb{C}^{N_s}$ denote the baseband signal transmitted by the ith transmit

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antenna, where N_s is the code length of transmit waveform. The transmit signal matrix can be denoted by $\mathbf{S}=\left[\mathbf{s}_1,\mathbf{s}_2,\cdots,\mathbf{s}_{N_t}\right]^{\mathrm{T}}\in\mathbb{C}^{N_t\times N_s}$. For convenience, assume that a uniform linear array (ULA) and a half-wavelength spacing between adjacent antennas are used both for transmitting and receiving. The waveform illuminated into spatial direction θ (i.e. angular waveform) can be expressed as [11]

$$\mathbf{x}(f)=\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S} \tag{1}$$

where $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose operation, $\mathbf{a}_t(f)$ is the transmitting steering vector, $f=0.5\sin(\theta)$ is defined as the normalized angular frequency. Note that the angular waveform is not constant modulus except θ is equal to zero.

The transmit beampattern (i.e. the power distribution of transmit waveform in the spacial domain) is defined as [11]

$$\begin{aligned}p(f)&=\mathbf{x}(f)\mathbf{x}^{\mathrm{H}}(f)/N_s=\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)/N_s\\&=\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)/N_s\end{aligned} \tag{2}$$

where $\mathbf{R}_0=\mathbf{S}\mathbf{S}^{\mathrm{H}}$ is waveform covariance matrix. The spacial auto correlation function of angular waveform $\mathbf{x}(f)$ at *k*th shift is written as

$$r_k(f)=\frac{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{J}_k\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)}=\frac{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{J}_k\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)} \tag{3}$$

where $\mathbf{J}_k$ is the shift matrix, i.e.,

$$\mathbf{J}_k=\mathbf{J}_{-k}^{\mathrm{T}}=\begin{bmatrix}\mathbf{0}_{(N_s-k)\times k} & \mathbf{I}_{N_s-k}\\ \mathbf{0}_{k\times k} & \mathbf{0}_{k\times(N_s-k)}\end{bmatrix},0\le k\le N_s. \tag{4}$$

Note that the denominator in (3) is used for normalization. From (4), it is easily proved that the $r_k(f)$ is conjugate symmetric with respect to $k=0$, i.e.,

$$\begin{aligned}r_{-k}(f)&=\frac{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{J}_{-k}\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)}=\frac{\left[\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{J}_k\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f)\right]^{\mathrm{H}}}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)}\\&=r_k^*(f)\end{aligned}. \tag{5}$$

Hence, we have $\left|r_k(f)\right|=\left|r_{-k}(f)\right|$. In addition, the spacial cross correlation of angular waveforms $\mathbf{x}(f)$ and $\mathbf{x}(f')$ at *k*th shift can be expressed as

$$r_k(f,f')=\frac{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}\mathbf{J}_k\mathbf{S}^{\mathrm{H}}\mathbf{a}_t(f')}{\sqrt{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)}\sqrt{\mathbf{a}_t^{\mathrm{H}}(f')\mathbf{R}_0\mathbf{a}_t(f')}}. \tag{6}$$

Similar to (5), it can be proved that $r_{-k}(f,f')=r_k^*(f',f)$. Thus, we can obtain $\left|r_{-k}(f,f')\right|=\left|r_k(f',f)\right|$.

For colocated MIMO system, the synthesis angular waveform in space often is not constant modulus. However, in real applications, the amplifiers of transmit circuits of jammers often operate in the saturation mode. Hence, when the angular waveform is intercepted, the DRFM repeat jammer need to modulate it and then retransmit a constant modulus waveform. For pulse compression radar, a matched filter often is used in the receive end.

Hence, to maintain a highly similarity with the intercepted signal, the DRFM repeat jammers need to accurately extract the phase information of the intercepted signal and then retransmit it after scaling and delaying. Specifically, when the jammer intercepts the angular waveform $\mathbf{x}(f)$, the jamming signal can be written as

$$\mathbf{s}_J(f)=\alpha\exp\left(\mathrm{j}\boldsymbol{\varphi}(f)\right) \tag{7}$$

where α is a real parameter ruling the jamming intensity and $\boldsymbol{\varphi}(f)$ denotes the phases (in radians) of $\mathbf{x}(f)$, i.e.,

$$\boldsymbol{\varphi}(f)=\arg\left(\mathbf{x}(f)\right) \tag{8}$$

where $\arg(\cdot)$ is the argument operation.

Generally, for DRFM repeat jammer, it is possible to construct false targets in both positive and negative range offsets by properly delaying the intercepted signal. However, the accurate delay time is hard to know for radar. To weaken the effect of DRFM repeat jammer, it requires low cross correlation level between angular waveform and jamming signal, which contains two parts. Firstly, when angular frequency $f=f'$, the cross correlation function of $\mathbf{x}(f)$ and $\mathbf{s}_J(f)$ at *k*th shift can be expressed as

$$J_k(f)=\frac{\mathbf{s}_J(f)\mathbf{J}_k\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)}. \tag{9}$$

Secondly, when angular frequency $f\ne f'$, the cross correlation function of $\mathbf{x}(f)$ and $\mathbf{s}_J(f')$ at *k*th shift is

$$J_k(f,f')=\frac{\mathbf{s}_J(f')\mathbf{J}_k\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{S}}{\mathbf{a}_t^{\mathrm{H}}(f)\mathbf{R}_0\mathbf{a}_t(f)} \tag{10}$$

B. Problem Formulation

To achieve against repeat radar jammer, the following goals need to accomplish.

1) *Transmit beampattern design*: To make the transmit power focus the directions of interest, we need to design the transmit beampattern. Assume that the normalized angular frequency range [-0.5, 0.5] is uniformly dispersed into N_d frequencies, i.e., $f_i, i=1,2,\cdots,N_d$. Based on (2), the transmit beampattern can be denoted by

$$\mathbf{b}_s=\left[p(f_1),p(f_2),\cdots,p(f_{N_d})\right]^{\mathrm{T}}. \tag{11}$$

We design the transmit beampattern by approximating a desired beampattern $\mathbf{b}_{\mathrm{d}}$, which mathematically denotes $\left|\mathbf{b}_{\mathrm{s}}-\gamma\mathbf{b}_{\mathrm{d}}\right|$, where $\gamma \geq 0$ is a scaling factor.

2) *Autocorrelation sidelobes suppression*: To easily identify the true targets, the angular waveform should have low auto correlation sidelobes. Assume that there are N_{a} normalized angular frequencies of interest, i.e, $\mathbf{f}_{\mathrm{a}}=\left\{f_{\mathrm{a}1}, f_{\mathrm{a}2}, \cdots, f_{\mathrm{a}N_{\mathrm{a}}}\right\}$. Hence, we need to suppress the autocorrelation sidelobes of the N_{a} angular waveforms. To obtain low range sidelobes, we resort to suppressing APSL. From (5), the APSL of the N_{a} angular waveforms can be expressed as

$$\mathrm{APSL}=\max_{\substack{k=1,2,\cdots,(N_{\mathrm{s}}-1) \\ f_{\mathrm{a}m}\in\mathbf{f}_{\mathrm{a}},\, m=1,2,\cdots,N_{\mathrm{a}}}}\left|r_k\left(f_{\mathrm{a}m}\right)\right|. \tag{12}$$

3) *Cross correlation sidelobes suppression*: To weaken the influences between angular waveforms from different angular frequencies, the cross correlation sidelobes should be as low as possible. For the N_{a} angular frequencies of interest, there are N_{c} ($N_{\mathrm{c}}=N_{\mathrm{d}}\left(N_{\mathrm{d}}-1\right)$) combinations of different angular frequencies, i.e,

$$\mathbf{\Omega}=\left\{\left(f_{\mathrm{a}1}, f_{\mathrm{a}2}\right),\left(f_{\mathrm{a}1}, f_{\mathrm{a}3}\right),, \cdots,\left(f_{\mathrm{a}(N_{\mathrm{a}}-1)}, f_{\mathrm{a}N_{\mathrm{a}}}\right)\right\}. \tag{13}$$

Similar to (12), the PCCL can be denoted by

$$\mathrm{PCCL}=\max_{\substack{k=0,\pm1,\pm2,\cdots,\pm(N_{\mathrm{s}}-1) \\ \left(f_{\mathrm{a}n}, f_{\mathrm{a}n}^{'}\right)\in\mathbf{\Omega},\, n=1,2,\cdots,N_{\mathrm{c}}}}\left|r_k\left(f_{\mathrm{a}n}, f_{\mathrm{a}n}^{'}\right)\right|. \tag{14}$$

4) *Jamming signal suppression*: To reject the DRFM repeat jammer, the jamming cross correlation sidelobes between angular waveforms and jamming signals should be suppressed. Firstly, the JPCCL of cross correlation function of $\mathbf{x}(f)$ and $\mathbf{s}_{\mathrm{J}}(f)$ can be written as

$$\mathrm{JPCCL}_{\mathrm{cr}1}=\max_{\substack{k=0,\pm1,\pm2,\cdots,\pm(N_{\mathrm{s}}-1) \\ f_{\mathrm{a}m}\in\mathbf{f}_{\mathrm{a}},\, m=1,2,\cdots,N_{\mathrm{a}}}}\left|J_k\left(f_{\mathrm{a}m}\right)\right| \tag{15}$$

Secondly, the JPCCL of the cross correlation function of $\mathbf{x}(f)$ and $\mathbf{s}_{\mathrm{J}}\left(f^{'}\right)$ is expressed as

$$\mathrm{JPCCL}_{\mathrm{cr}2}=\max_{\substack{k=0,\pm1,\pm2,\cdots,\pm(N_{\mathrm{s}}-1) \\ \left(f_{\mathrm{a}n}, f_{\mathrm{a}n}^{'}\right)\in\mathbf{\Omega},\, n=1,2,\cdots,N_{\mathrm{c}}}}\left|J_k\left(f_{\mathrm{a}n}, f_{\mathrm{a}n}^{'}\right)\right| \tag{16}$$

Based on (15) and (16), the JPCCL for the MIMO radar system can be defined as

$$\mathrm{JPCCL}=\max\left[\mathrm{JPCCL}_{\mathrm{cr}1}, \mathrm{JPCCL}_{\mathrm{cr}2}\right]. \tag{17}$$

Combining (12), (14) and (17), the appearing PSL among angular waveforms and jamming signals can be expressed as

$$\mathrm{PSL}=\max\left[\mathrm{APSL},\ \mathrm{PCCL},\ \mathrm{JPCCL}\right]. \tag{18}$$

Based on the four properties mentioned above, we formulate the optimization problem as follows:

$$\begin{aligned} &\min_{\gamma,\mathbf{\Phi}}\ \left[\mathrm{PSL};\ \beta\left|\mathbf{b}_{\mathrm{s}}-\gamma\mathbf{b}_{\mathrm{d}}\right|\right] \\ &\text{s.t.}\quad \gamma\geq 0 \end{aligned} \tag{19}$$

where $\mathbf{\Phi}$ is the phase of transmit signal matrix, $|\cdot|$ denotes the absolute value and $\beta \geq 0$ is the weight for the tradeoff between the range sidelobes and beampattern matching degree. Indeed, the larger the β is, the better the matching degree between the transmit beampattern and desired beampattern, however, the worse the PSL is. In practice, we can adjust β to make a tradeoff.

Due to the constant modulus constraint of transmit waveform, (19) is a nonconvex optimization problem, which can be solved by many existing algorithms, such as the simulated annealing (SA) algorithm, the genetic algorithm (GA) and the sequential quadratic programming (SQP) algorithm. But none of them can find the global minimum. In this paper, we exploit the SQP algorithm to address (19).

III. NUMERICAL RESULTS

In this section, we devote to evaluating the performance of the proposed method. Throughout the simulations, unless otherwise explicitly stated, consider a MIMO radar system equipped with ULA and inter-spacing of half a wavelength. The transmit antenna number is equal to the receive antenna number, i.e, $N_{\mathrm{t}}=N_{\mathrm{r}}$. We set the code length of transmit waveform $N_{\mathrm{s}}=128$, the parameter in (7) $\alpha=1$ and the weight coefficient in (19) $\beta=0.01$. The normalized angular frequency range [-0.5,0.5] is uniformly dispersed into $N_{\mathrm{d}}=101$ frequencies, i.e., the sample interval is 0.01. Consider three interested directions located in $\mathbf{f}_{\mathrm{a}}=\left\{f_{\mathrm{a}m}=-0.3, 0, 0.3\right\}$, and the desired beampattern is

$$\mathbf{b}_{\mathrm{d}}(f)=\begin{cases}1, & f\in\left[f_{\mathrm{a}m}-0.05, f_{\mathrm{a}m}-0.05\right], m=1,2,3 \\ 0, & \text{otherwise}\end{cases}. \tag{20}$$

Note that since the problem (19) is a nonconvex optimization problem, the initial value has effects on the final optimization result. Hence, we randomly provide 20 initial values to the SQP algorithm for each subsequent optimization and choose the best solution as the final result.

The design transmit beampatterns with different antenna numbers are depicted in Fig. 1 together with the desired beampattern. Fig. 1 shows that the design transmit beampatterns are close to the desired beampattern. Besides, we also can see that increasing the antenna number can result in better approximation. This is due to the fact that increasing the antenna number is tantamount to increasing the degrees of freedom (DOF) to optimize.

The autocorrelation sidelobes and cross correlation sidelobes of the three angular waveforms with $N_{\mathrm{t}}=12$ are drawn in Fig. 2 and Fig. 3, respectively. Moreover, the jamming cross correlation sidelobes between the angular waveforms and corresponding jammer signals are plotted in Fig. 4. From Figs. 2-4, it is observed that on one hand, the auto/cross correlation sidelobes of the angular waveforms are effectively suppressed, and on the other hand, the repeat jammer signals are also suppressed. Hence, the true targets can be easily recognized.

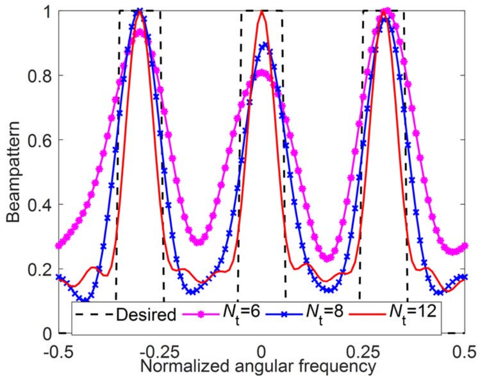

Fig. 1 Comparison of the design beampattern for different N_t

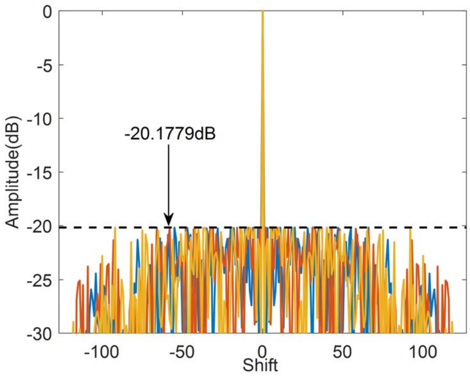

Fig. 2 Auto correlation result of angular waveforms

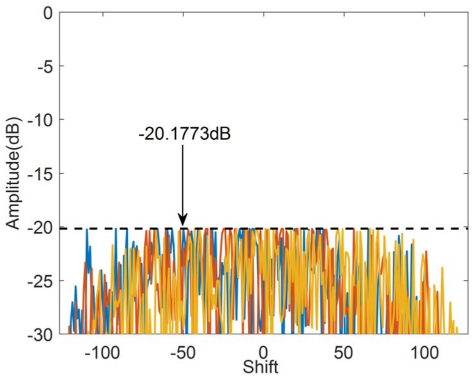

Fig. 3 Cross correlation result of different angular waveforms

In the following, the effects of parameter α in (7) on PSLs (see (18)) are investigated in Fig. 5 with $N_t = 12$ and $\beta = 0$. Fig. 5 highlights that the PSL decreases gradually with the α increasing. This is reasonable because increasing α means increasing the jamming intensity. In practice, to penalize or decrease strong jamming, we can resort to enhancing the complexity of transmit waveform, such as adding phase perturbation of an LFM chirp signal [6].

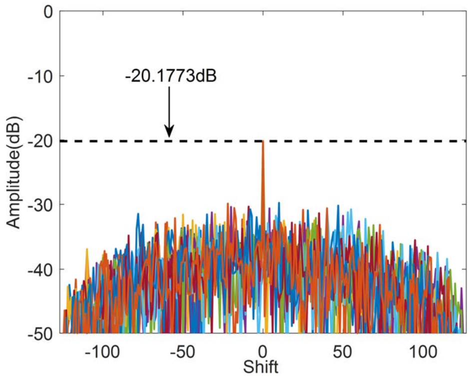

Fig. 4 Cross correlation result between angular waveforms and jamming signals

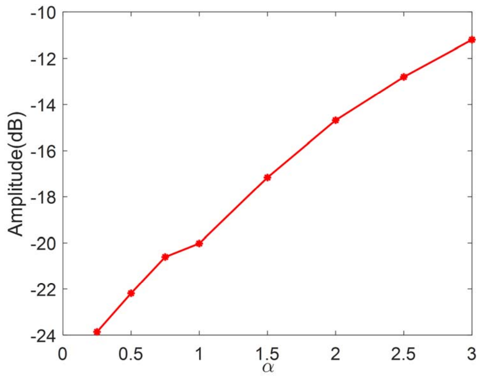

Fig. 5 PSLs of the proposed method for different α

IV. Conclusion and Discussion

In this paper, we consider a colocated MIMO radar system and propose an approach to combat the DRFM repeat jammer where the specific delay time information is unknown. When the jammers capture angular waveforms (nonconstant modulus), the repeat jamming signal should be constant modulus. As such, we formulate a min-max design criterion by matching desired beampattern, suppressing the APSL and PCCL of angular waveforms and the JPCCL between the angular waveforms and repeat jamming signals. The design criterion is a nonconcex problem and we address it by the SQP algorithm. Numerical results verify the effectiveness of the proposed method.

In practice, to further suppress the PSL, the received beamforming can be taken into account to reduce the cross correlation sidelobes of different angular waveforms, leaving more DOFs to suppress the other component in (19). Meanwhile, the clutter suppression can be taken in account. It merits further study in this case.

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