Sparse Clutter estimation for STAP based on Decouple Atomic Norm minimization

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***Abstract*—In this paper, a novel STAP algorithm based on Decouple Atomic Norm minimization is proposed, the new algorithm is able to get more accurate support set and amplitude estimation in sparse recovery operation. Meanwhile the method is with low computation complexity compared with other sparse recovery methods, which show a main advantage in sparse clutter spatial-temporal spectrum estimation.**

Keywords—STAP; atomic norm; Sparse recovery;

I. INTRODUCTION

Space-Time Adaptive Processing (STAP) is a widely used technique in airborne radar signal processing which was originally developed for clutter suppressed in moving targets detection [1]. The key step of STAP is to obtain the estimation of clutter covariance matrix $\boldsymbol{R}_C$ [2][3], which is implemented via sampling the echo in the concerned area in targets free scene. Conventional methods for estimating $\boldsymbol{R}_C$ are mainly relied on the maximum likelihood estimation via averaging the data of received echoes from the several range bin adjacent to the detection range cell. Recently, the compressive sensing theory [4][5] has drawn great attention in STAP, since the strong clutter spectrum mainly concentrated on clutter-ridge line in the spatial-temporal plane [6]. High-resolution spatial-temporal spectrum estimation can be obtained with only a few snapshots via sparse recovery method. However, these sparse recovery methods are mainly based on L1-norm minimum algorithm, in order to obtain a more accuracy estimation of the clutter and noise, more refined grids of spatial and Doppler frequencies should be drawn to overcome the basis mismatched problem. This will greatly increase the computation burden for L_1 norm recovery. The atomic norm minimization (ANM) for continuous parameters recovery was mainly used to sparse frequency recovery to solve the off grid problem which leads a potential for deal with the issue.

In this paper, the STAP signal model is turned into a grid-less sparse recovery model and is solved via a Decoupled ANM method to complete the estimation of clutter spatial-temporal spectral.

II. PROBLEM FORMULATION

Consider a side-looking airborne radar [1] with an uniform linear array (ULA) that contain M antenna elements and each antenna transmits N coherent pulses (with T as theirs pulse repetition interval). Let $\boldsymbol{x}$ denote the vectorized received data of a given range Cell Under Test (CUT). Then it can be expressed as

$$\boldsymbol{y} = \boldsymbol{x}_t + \boldsymbol{x}_c + \boldsymbol{n} \tag{1}$$

where, $\boldsymbol{x}_t$, $\boldsymbol{x}_c$ and $\boldsymbol{n}$ are the target signal ,the clutter and the noise, respectively. Clearly,

$$\boldsymbol{x}_t = \alpha \boldsymbol{a}_{(f_{st}, f_{dt})} \tag{2}$$

where α is the amplitude of the target and (f_{st}, f_{dt}) denote the normalized spatial and Doppler frequencies of the target, and $\boldsymbol{a}_{(f_{st}, f_{dt})}$ is the spatial-temporal steering vector which can be expressed as

$$\boldsymbol{a}_{(f_{st}, f_{dt})} = \boldsymbol{a}_{(f_{st})} \otimes \boldsymbol{a}_{(f_{dt})} \tag{3}$$

where

$$\boldsymbol{a}_{(f_{st})} = [1, \mathrm{e}^{-jf_{st}}, ..., \mathrm{e}^{-j(M-1)f_{st}}]^T \in \mathbb{C}^M$$
$$\boldsymbol{a}_{(f_{dt})} = [1, \mathrm{e}^{-jf_{dt}}, ..., \mathrm{e}^{-j(N-1)f_{dt}}]^T \in \mathbb{C}^N$$

is the separated steer vector about spatial and Doppler frequencies.

According to [2] the clutter is modeled as following

$$\boldsymbol{x}_c = \sum_{i=1}^{Nc} \sigma_i \, \boldsymbol{a}_{(f_{si},\, f_{di})} = \sum_{i=1}^{Nc} \sigma_i \, \boldsymbol{a}_{(f_{si})} \otimes \boldsymbol{a}_{(f_{di})} \tag{4}$$

where, σ_i is the complex echo strength of the *i*-th clutter patch, N_c is the total number of clutter pitch of a given CUT.

In conventional STAP framework the optimal adaptive filter weight is set according to the criterion of maximum the output signal-interference-ratio (SINR), and can be written as

$$\boldsymbol{w}_{opt} = \boldsymbol{R}^{-1} \boldsymbol{a}_{(f_{st}, f_{dt})} \tag{5}$$

where $\boldsymbol{R} = \boldsymbol{R}_C + \boldsymbol{R}_n$ is sum of the covariance of clutter and noise.

Therefore, in strong clutter scenario, the key step for successfully implementing STAP is to estimate the clutter covariance matrix (CCM) $\boldsymbol{R}_C \in \mathbb{C}^{MN}$, which is

$$R_c = \boldsymbol{E}[\boldsymbol{x_c x_c^H}] = \sum_{i=1}^{Nc} E\{|\sigma_i|^2\}\boldsymbol{a}_{(f_{si},\, f_{di})}\boldsymbol{a}^H_{(f_{si},\, f_{di})} \tag{6}$$

It is noted that, to a given clutter patch, it holds that

$$\begin{cases} f_{si} = \cos\varphi_i \cos\theta_i \cdot d / \lambda \\ f_{di} = \beta \cdot f_{si} \quad , \quad \beta = 2vT / d \end{cases} \tag{7}$$

where, d is the inner space of the ULA, λ is the wavelength of the transmitted signal, φ_i , θ_i are the azimuth and elevation angles of the *i*-th patch of clutter, respectively, and v is the velocity of radar platform. Then, the structured property of clutter spectrum can be shown in Fig. 1 as follows.

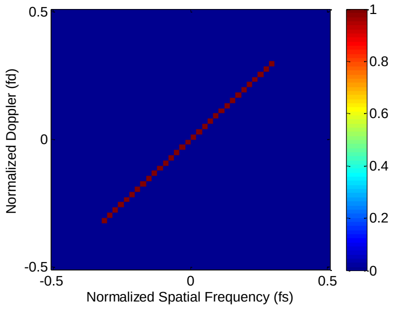

Fig. 1. The spatial-temporal spectrum example: clutter ridge when $\beta = 1$.

It can be seen that, in spatial-temporal plane, the clutter echo energy distributed concentrated in a line so called clutter ridge [6]. Previous studies have shown that the spatial-temporal is sparse [7], and the sparse recovery method can be utilized for STAP .

III. SPARSE RECOVERY METHOD FOR STAP

A. L1 norm- STAP

Once we discretize the spatial and Doppler frequencies into N_S and N_d grid points, then the clutter echo can be rewritten as

$$\boldsymbol{x}_c = \sum_{l=1}^{Nd}\sum_{m=1}^{Ns} \gamma_{l,k}\, \boldsymbol{a}_{(f_{sm},\, f_{dl})} = \boldsymbol{A\zeta} \tag{8}$$

where, $\boldsymbol{A} = [\boldsymbol{a}_{(f_{s,1},\, f_{d1})}, \boldsymbol{a}_{(f_{s,2},\, f_{d,1})}, ..., \boldsymbol{a}_{(f_{s,Ns},\, f_{d,Nd})}] \in \mathbb{C}^{MN \times NsNd}$ is the spatial-temporal steering dictionary and $\boldsymbol{\gamma} = [\gamma_{1,1}, \gamma_{1,2}, ..., \gamma_{Ns,Nd}]^T \in \mathbb{C}^{NsNd \times 1}$ denotes the vectored clutter strength in spatial and Doppler frequencies plane with its nonzero elements refer to the true location of the clutter patches where,

$$\gamma_{l,m} = \begin{cases} \sigma_i & (m,l) = (i,i) \\ 0 & else \end{cases} \tag{9}$$

Therefore $\boldsymbol{\gamma}$ is a sparse vector, which can be recovery via sparse recovery method.

In reality, the estimation of the clutter covariance is carried out via sampling the echo in the target free cell and the received echo can be written as,

$$\boldsymbol{y} = \boldsymbol{A\gamma} + \boldsymbol{n} \tag{10}$$

Where, $\boldsymbol{A}, \boldsymbol{\gamma}$ are the same as in (7) and (8) respectively and $\boldsymbol{n}$ is the noise. Thus, the clutter strength ζ can be estimated by solving the following sparse recovery problem

$$\hat{\boldsymbol{\gamma}} = \arg\min \|\boldsymbol{\gamma}\|_0 \quad s.t. \ \|\boldsymbol{x} - \boldsymbol{A\gamma}\|^2 \le \eta \tag{11}$$

where, η is the noise level.

With proper transformation the problem (11) can be reduced to a linear programming (LP) problem or Lasso [14] or L_1-norm threshold iterative method [4,5]. Then CCM can be achieved even with a single snapshot [7].

$$\begin{cases} p_i = |\gamma_i|^2 \\ \boldsymbol{R}_c = \boldsymbol{A} \bullet diag(p_1, p_2, ..., p_{NsNd}) \bullet \boldsymbol{A}^H \end{cases} \tag{12}$$

In reality multiple snapshots is often used. Assume that there are K snapshots of target free range cell, then the received data can be expressed as

$$\boldsymbol{Y} = \boldsymbol{A\Gamma} + \boldsymbol{N} \tag{13}$$

where $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_K]$ is the K snapshots received echo, $\boldsymbol{\Gamma} = [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, ..., \boldsymbol{\gamma}_K]$ is the strength vector of clutter with each $\boldsymbol{\gamma}_i$ own the same support, $\boldsymbol{N} = [\boldsymbol{n}_1, \boldsymbol{n}_2, ..., \boldsymbol{n}_K]$ is the noise. Then conducting the sparse recovery via snapshot wise and averaging the vector recovery to obtain a fine accuracy estimation of (CCM) via

$$p_i = \sum_{j=1}^{K} |\Gamma_{ij}|^2 / K \tag{14}$$

and (11).

B. Atomic Norm minimization for STAP

In order to obtain a more accuracy estimation of the clutter and noise, more refined grids of spatial and Doppler frequencies should be drawn to overcome the basis mismatched problem. This will greatly increase the computation burden for L_1 norm recovery. The atomic normal minimization ANM for continuous parameters recovery was mainly used to sparse frequency recovery to solve the off grid problem.

The atomic norm is defined as [9]

$$\|\boldsymbol{x}\|_{\mathcal{A}}=\inf_{\theta_k\in\Omega}\left\{\sum_k|c_k|\,\middle|\,\boldsymbol{x}=\sum_k c_k\boldsymbol{a}_{(\theta_k)},\boldsymbol{a}\in\mathcal{A}\right\}$$
$$\mathcal{A}=\ \boldsymbol{a}_{(\boldsymbol{\theta})}\,|\,\boldsymbol{a}_{(\boldsymbol{\theta})}=\boldsymbol{a}_{(f_s)}\otimes\boldsymbol{a}_{(f_d)}\ ,\boldsymbol{\theta}_k=(fs,fd)\in\Omega=[0,2\pi]\times[0,2\pi] \tag{15}$$

where Ω is a continuous parameter space. By definition the atomic norm aims to find out the vector with minimum L_1 norm of coefficient that can be represent by linear combination of $\boldsymbol{u}_{(\theta_k)}$ with different parameter θ_k from a continuous parameter space.

To solve (9) it can be attributed as following convex optimum problem

$$\{\boldsymbol{x}_c,\hat{\boldsymbol{u}}\}=\arg\min_{\boldsymbol{x}\,\boldsymbol{u}}\frac{1}{2}[tr\left(S(\boldsymbol{T})\right)+t+\lambda\|\boldsymbol{x}-\boldsymbol{y}\|^2]$$
$$s.t.\qquad\begin{bmatrix}S(\boldsymbol{T}) & \boldsymbol{x}\\ \boldsymbol{x}^H & t\end{bmatrix}\geq 0 \tag{16}$$

Where $S(\boldsymbol{T})\in\mathbb{C}^{MN\times MN}$ is a two fold block Toeplitz matrix as is defined in [10], $\lambda>0$ is the regularization coefficient, $\boldsymbol{y}$ is with the same definition as in (10). Then the sparse clutter can be obtained as $\boldsymbol{x}_c$.

C. Decoupled Atomic Norm minimization for STAP (DANM)

The main shortage of the vectored atomic norm minimum lie in its high computation complexity, i.e. the constraint in (16) is a semi-definite positive matrix with dimension of $(MN+1)\times(MN+1)$ which pose great challenge in STAP application when M,N are large enough.

Luckily borrow the idea from [11] we can decoupled the two dimension estimation problem into a SDP problem with constraint size decreases to $(M+N)\times(M+N)$ via new atomic norm which is defined as

$$\|\boldsymbol{X}\|_{\mathcal{A}}=\inf_{\theta_k\in\Omega}\left\{\sum_k|c_k|\,\middle|\,\boldsymbol{X}=\sum_k c_k\boldsymbol{A}_{(\theta_k)},\boldsymbol{a}\in\mathcal{A}\right\}$$
$$\mathcal{A}=\ \boldsymbol{A}_{(\boldsymbol{\theta})}\,|\,\boldsymbol{A}_{(\boldsymbol{\theta})}=\boldsymbol{a}_{(f_s)}\boldsymbol{a}^T_{(f_d)}\ ,\boldsymbol{\theta}\ =(fs,fd)\in\Omega=[0,2\pi]\times[0,2\pi] \tag{17}$$

here, the atoms $\boldsymbol{A}_{(\theta_k)}\in\mathbb{C}^{M\times N}$ is a set of matrices. It is easy to see that if

$$\boldsymbol{X}=\sum_k\sigma_k\boldsymbol{A}_{(\theta_k)}=\sum_k\sigma_k\boldsymbol{a}_{(f_s)}\boldsymbol{a}^T_{(f_d)} \tag{18}$$

then

$$\boldsymbol{x}=vec(\boldsymbol{X})=\sum_k\sigma_k\boldsymbol{a}_{(f_{dk})}\otimes\boldsymbol{a}_{(f_{sk})} \tag{19}$$

which is the same as in (4). Therefore the receiver echo in (10) can be turned into matrix model and rewritten as following

$$\boldsymbol{Y}=\boldsymbol{X}_c+\boldsymbol{N}$$
$$\boldsymbol{X}_c=\sum_k\sigma_{\boldsymbol{k}}\boldsymbol{A}_{(\theta_k)} \tag{20}$$

here, σ_k is the same as in (19) and $\boldsymbol{A}_{(\theta_k)}$ is defined in (17), thus the clutter estimation problem can be solved via

$$\{\boldsymbol{X}_c,\hat{\boldsymbol{u}},\hat{\boldsymbol{v}}\}=\arg\min_{\boldsymbol{X}\,\boldsymbol{u}\,\boldsymbol{v}}\frac{1}{2}[tr\left(T(\boldsymbol{u})+T(\boldsymbol{v})\right)+\lambda\|\boldsymbol{X}-\boldsymbol{Y}\|_F^2]$$
$$s.t.\qquad\begin{bmatrix}T(\boldsymbol{u}) & \boldsymbol{X}\\ \boldsymbol{X}^H & T(\boldsymbol{v})\end{bmatrix}\geq 0 \tag{21}$$

where, $T(\boldsymbol{u})$ denote the Hermite Toeplitz matrix generated from vector $\boldsymbol{u}$, $\|*\|_F$ denotes the Frobenius norm and $\lambda>0$ is the regularization coefficient. Then according to theorems in [11] the sparse clutter $\boldsymbol{X}_c$ can be estimated as $\boldsymbol{X}_c$ from (21) via cvx tool pack [12].

Then the the adaptive filter step, we aims at cancelling both the clutter and interference while preserving the target from look-direction. By suitable choosing dimension of sub-block of spatial-temporal data, we turn to solving following equartion

$$\begin{bmatrix}\boldsymbol{a}_t\\ X_{sub}\end{bmatrix}\cdot\boldsymbol{w}_{sub}=\begin{bmatrix}c\\ \boldsymbol{0}\end{bmatrix} \tag{22}$$

where, $\boldsymbol{a}_t$ is the target spatial-temporal steer vector, X_{sub} is the data sub-block of $\boldsymbol{X}_c$, $\boldsymbol{w}_{sub}$ is the filter weight vector.

As for multiple snapshots cases, it can be obtained a more precise estimation via averaging the result of each snap which is implemented in similar way that from (12) to (15).

IV. Numerical Simulations

In this section, experiments are conducted to show the efficiency of the ANM-STAP method compare with other methods estimation L1-norm recovery. Among all experiments, the antenna is ULA with M=8 elements. And Coherent pulse number is N=8 . The number of clutter cells in scene is set to $N_c=4$, and the clutter cells uniformly distributed in Normalized spatial frequency region $[-0.2,0.2]$. The clutter strength of each cells are set to be i.i.d. Complex Gaussian distribution i.e., $CN(1,0.1)$. While the noise is zero-mean Complex Gaussian distribution. The Clutter-to-Noise Ratio (CNR) is set to 15 dB.

Fig.2 shows us the spatial-temporal spectrum estimation. All results are gotten from an average of conducted in 10 trials. For each sub figure, the horizontal axis is for Normalized spatial frequency and yaxis is for normalized Doppler frequency. The value of each spatial-temporal point indicates the power of the scatter pitch. Fig.2(a) shows the true clutter spectrum of the simulation data. Fig.2(b) shows the L1-norm STAP estimation with total number of grids KK=256 Fig.2(c) is the L1-norm STAP estimation with total number of grids KK=1024. Fig.2(d) is the result of the proposed DANM-STAP method. It can be seen that the L1-norm STAP will result false clutter cells and with the grids more refined the better the location recovery performance of clutter cells is, but the amplitude recovery error is still remarkable. While the ANM-STAP method is able to successfully recovery both the location and the complex amplitude of clutter cells.

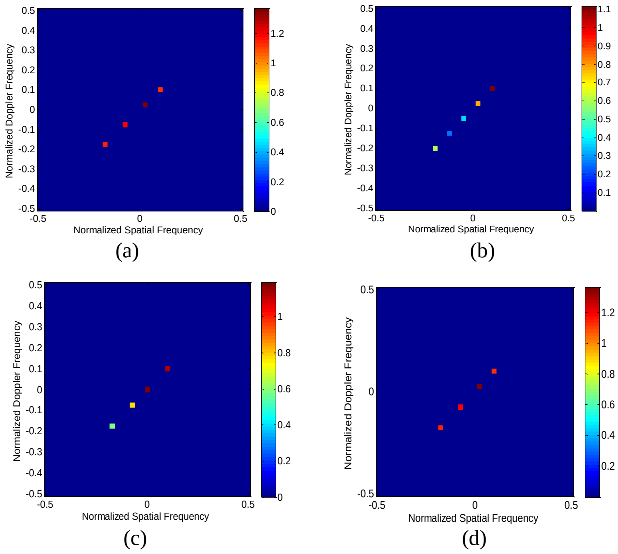

Fig. 2. Normalized spatial-temporal spectrum estimation.(a) the original clutter (b) L1-norm STAP estimation with KK=256 grids (c) L1-norm STAP estimation with KK=1024 grids (d) DANM-STAP estimation.

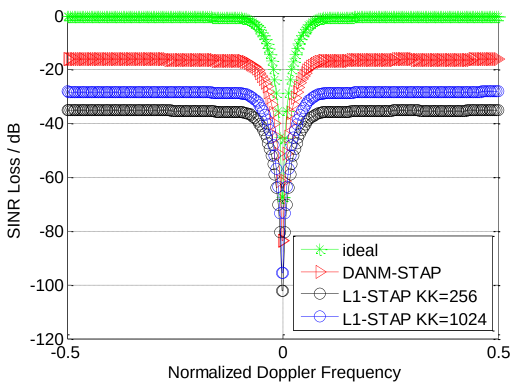

Fig. 3. SINR Loss performance of each method compared with ideal clutter supressed filter.

Fig. 3 shows the comparison of normalized SINR Loss performance between L1-norm STAP estimation with KK=256 grids, L1-norm STAP estimation with KK=1024 grids, DANM-STAP estimation and the ideal clutter suppressed filter (the covariance of clutter is true). In this figure, the performance of SINR Loss of each method is normalized, the horizontal axis is normalized Doppler frequency $f_d = 0$. The target angle in spatial frequency is set to be zero. So the strongest clutter appears in $f_d = 0$. As illustrated in Fig. 2, the performance of the SINR Loss of DANM-STAP in clutter suppressed is nearly 10 dB better than L1-norm STAP estimation with KK=1024 grids, and 17 dB than L1-norm STAP estimation with KK=256 grids, this mean the DANM-STAP will get a higher target detection performance.

TABLE I. TIME CONSUMING FOR EACH METHOD (SECOND)

DANM-STAP	L1-norm STAP KK=256	L1-norm STAP KK=1024
8.121	17.246	46.514

In the end we show the time consuming performance on a PC (Intel Core i5-2500, 3.3GHz) for each method in table 1, it can be seen that the DANM-STAP is able to estimate the sparse clutter much faster than L1-norm STAP.

V. CONCLUSION

In this paper, we propose a novel STAP algorithm based on Decouple Atomic Norm minimization, the new algorithm is able to get more accurate support set and amplitude estimation in sparse recovery operation. Meanwhile the method is with low computation complexity compared with other methods, which is a main advantage in sparse spatial-temporal spectrum estimation and lead a potential for deal the clutter estimation in STAP when the number of array elements and coherent pulses are very large. However, in reality the clutter environments is more complex, the estimation of sparse clutter spatial-temporal spectrum need to be further studied.

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