Rocket detection using passive radar – challenges and solutions

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Abstract—In the paper selected aspects of rocket detection with the use of passive radar are analyzed. The bistatic radar cross-section of a rocket is calculated using electromagnetic simulator. Next, the radar equation for passive radar is analyzed taking into consideration the radiation patterns of the transmitter antenna. Limitations of integration time are also discussed. The theoretical analysis is supplemented with measurement results – the processing results of a rocket observed with a DVB-T based passive radar are shown and analyzed.

I. INTRODUCTION

Passive radar is used primarily for detection of airborne targets, mainly aircraft [1], [2], [3]. Other types of targets of interest include cars, helicopters, pedestrians and ships [4], [5], [6], [7]. Passive radar is also used for target imaging [8], [9], [10] and ionospheric observations [11]. In this paper we focus on using passive radar for observation of rockets [12], [13]. The rockets under consideration are amateur experimental and hobby rockets launched from the ground surface. Typical length of such a rocket is from 1 m to 3 m. Both sub- and super-sonic rockets are of interest.

Rocket detection detection and tracking is a challenging task. First problem is the radar cross-section (RCS) of the rocket. It is not as small as in case of small drones, but not as large as of passenger aircraft. Rocket detection can be also significantly influenced by the transmitter illumination as the target can fall in the null of the transmitter radiation pattern. Other challenge is high maneuverability of the target. Rockets tend to move very fast and accelerate rapidly. From this point of view, the classical signal processing algorithms applied in case of other types of targets, may prove to be inadequate in case of rockets.

In the first part of the paper we analyze the RCS of the rocket using electromagnetic solver (CST Studio). Next, the radar equation for passive radar is reviewed with special emphasis on the radiation patterns of the transmitter antenna. At the end of the paper, results of measurements are presented showing some of the typical problems encountered during detection of rockets.

II. DETECTION RANGE

A. RCS analysis

The RCS calculations for the rocket were performed using CST (Computer Simulation Technology) Studio software [14].
and
minimum detectable RCS
B. Range equation
where
Fig. 3. 3D visualization of calculated RCS

where
r is the cylinder radius, h is the cylinder height and \( \lambda \)

is the wavelength. For the considered rocket case \( r = 6 \) cm

and \( h = 270 \) cm, which results in value of \( \sigma_{\text{max}} \)

equal to 7.94 \( \text{dBsm} \) (6.2 \( \text{m}^2 \)) for frequency of 680 MHz. This is very

close to the result obtained from the CST Studio simulation,

approximately equal to 8 \( \text{dBsm} \) for back scattering (monostatic

case).

Fig. 3 shows full 3D pattern. The observed shaped is highly

symmetrical with respect to the z-axis. For this reason the

azimuth cross-sections of the patterns are almost uniform. This

results form the fact that the rocket is highly symmetrical with

respect to the z-axis (the only exception are four stabilizers at

the end of the rocket).

Concluding the RCS simulations it can be stated that the

RCS of a rocket observed from the boresight is relatively high

– almost 10 \( \text{dBsm} \) in the considered case. When the rocket is

observed from the bottom or the top, the RCS is substantially

reduced – to the value of \(-10 \text{dBsm}\).

B. Range equation
Consider bistatic range equation, rearranged to express the

minimum detectable RCS \( \sigma_{\text{det}} \) [2]:

\[
\sigma_{\text{det}} = \frac{\text{SNR}_{\text{min}} (4\pi)^3 R_1^2 R_2^2 N_f k T_r L}{P_t G_t(\phi, \theta) G_r \lambda^2 T}
\]

(2)

where \text{SNR}_{\text{min}} is the minimum signal-to-noise ratio required

detection, \( R_1 \) is the transmitter-target range, \( R_2 \) is the

target-receiver range, \( N_f \) is the receiver noise figure, \( k \) is the

Boltzmann constant, \( T_r \) is the receiver noise temperature, \( L \)

are the system losses, \( P_t \) is the maximum transmitted power,

\( G_t(\phi, \theta) \) is the transmit antenna radiation pattern, \( G_r \)

is the receiver antenna gain \( \lambda \) is the wavelength and \( T \) is the

integration time. It was assumed that the receiver bandwidth

and signal bandwidth are the same, hence there is no bandwidth

dependency in (2) (the bandwidth in integration gain \( BT \) and

noise power \( kT_r B \) cancel each other).

For simplicity, often the transmitter antenna radiation pat-

tern is assumed to be omnidirectional. In practice, however,

this is far from true, and can significantly impact the radar

performance. In case of azimuth plane, transmitters are often

designed to radiate approximately with the same strength

in all directions, therefore the assumption on omidirectional

radiation is justified. However, in the elevation plane the beam

is relatively narrow, which allows the radiated power to be

focused in the direction of receivers that are placed on the

ground. Typically, the antennas are built using several radiating

elements stacked in the vertical direction. Based on those

assumptions, a simple model for radiation pattern can be

formulated as:

\[
G_t(\phi, \theta) = \frac{\sin \left( \frac{\pi L_v}{\lambda} \sin(\theta) \right)}{\pi L_v^2 \sin(\theta)}
\]

(3)

where \( \phi \) is the azimuth angle, \( \theta \) is the elevation angle (0\(^\circ\)

corresponds horizontal direction here), \( L_v \) is the vertical size

of the antenna. The pattern (3) is omnidirectional in the

azimuth direction, and has \( \sin(x)/x \) shape typical for apertures

with rectangular current distribution. Typically the elements

are spaced by \( \lambda/2 \), then the length of the antenna can be

recalculated to effective aperture length \( L_v = (N_v - 1)\lambda/2 \),

where \( N_v \) is the number of radiating elements.

In Fig. 4 the azimuth cross-section of the detectable RCS

calculated using (2) with transmitter radiation pattern (3) is

plotted for different altitudes of the target. The calculations

were performed for a representative set of DVB-T-based pas-

sive radar: \text{SNR}_{\text{min}} = 12 \text{ dB}, N_f = 10 \text{ dB}, T_r = 290 \text{ K},

L = 10 \text{ dB}, P_t = 100 \text{ kW}, G_r = 5 \text{ dBi}, f_c = 600 \text{ MHz}

and \( T = 50 \text{ ms} \). It was assumed that the antenna consists of

\( N_v = 16 \) vertical radiating elements with \( \lambda/2 \) spacing.

The transmitter is assumed to be at the ground level and the

elevation tilt of the antenna is 0\(^\circ\). The black contour indicates

the level of 0 \( \text{dBsm} \). The rings surrounding the transmitter

correspond to nulls of the elevation radiation pattern. The

size and width of the rings increases with target altitude.

This results from the fact that the same angular width of the

pattern lobe corresponds to larger distance at higher altitudes.

The shape of the ring is not symmetrical with respect to the

transmitter, as the detectable RCS is influenced by the Cassini

ovals – shapes characteristic for the bistatic range equation

(which result from the \( R_1^2 R_2^2 \) term in the bistatic equation).

In Fig. 5 the elevation cross-section of the detectable RCS

is shown. The observed fan shape corresponds to the elevation

radiation pattern. As can be seen, in the presented example

around 3 km above the receiver a null can be expected.

Presented results clearly indicate that when the observed

target is changing its altitude, the detection capabilities of the

radar can vary significantly. Even if the target is flying at
a constant altitude, is can fall into a null of the transmitter radiation pattern, and that will prevent it from being detected. In case of a rocket launched from the ground and flying straight up this can mean that the rocket will disappear from the radar at certain altitudes and geographical coordinates.

III. NUMERICAL RESULTS

In this section we show results of processing of measured data collected during observation of a rocket launch. The rocket under consideration is shown in Fig. 6. The rocket name is Strega and it had been designed by rocket enthusiast Damian Mayer. The rocket is 2.3 m in length and 8084 mm in diameter. It has three stabilizers. Everything except the rocket nose/head is made from aluminum 6060 alloy.

The data was collected using DVB-T-based passive radar. The transmitter used for target illumination was operating at the frequency of 634 MHz in horizontal polarization. The transmitter was located 77 km from the receiver. The launch site was located 200 m from the receiver. The rocket was flying almost vertically. The fact that the transmitter was far away from the launch site resulted in rocket being illuminated from the boresight, similar as in the electromagnetic simulations of RCS. The fact that the launch site was close to the receiver means that as the rocket climbed, the elevation angle $\theta$ at which the rocket was observed by the radar approached 180° rapidly (compare Fig. 2).

In Fig. 7 the crossambiguity function is shown with rocket echo visible. Each of the plots corresponds to different integration time, equal to 25 ms, 50 ms, 100 ms and 200 ms, respectively. The bistatic velocity resolution $\Delta V$ depends on
the integration time $T$ according to:

$$\Delta V = \frac{\lambda}{T} \quad (4)$$

For this reason increase in the integration time corresponds to proportional decrease of the velocity resolution. In the figure this can be seen as the echo becomes narrower in case of 50 ms integration time in comparison to 25 ms integration time. If the integration time is increased too much, the echo starts to spread across velocity because the target is accelerating. This can be visible in the last plot for integration time equal to 200 ms.

The presented result suggests that target acceleration can be a serious problem when longer integration time are desired. This is only partially true. If a traditional crossambiguity function is calculated that does not take target acceleration into account (see (5)), echo spreading and thus power loss can be expected.

$$\Psi(R, V) = \frac{T}{2} \int_{-T/2}^{T/2} x_e(t) \cdot x_r^*(t - \frac{R}{c}) \cdot \exp\left(\frac{2\pi \lambda}{\lambda} V t \right) \exp\left(\frac{j 2\pi \lambda}{\lambda} V t \right) dt \quad (5)$$

where $R$ is the bistatic range, $V$ is the bistatic velocity, $x_e(t)$ is the reference signal and $x_e(t)$ is the surveillance signal.

If, on the other hand, extended crossambiguity function is applied \cite{16}, \cite{17}, \cite{18}, \cite{19}, increase in integration time can not only provide higher echo power, but also information on target acceleration can be obtained. The crossambiguity function definition can be extended in the following way:

$$\Psi_A(R, V, A) = \frac{T}{2} \int_{-T/2}^{T/2} x_e(t) \cdot x_r^*(t - \frac{R}{c}) \cdot \exp\left(\frac{2\pi \lambda}{\lambda} \left( V t + \frac{A t^2}{2} \right) \right) dt \quad (6)$$

where $A$ is the bistatic acceleration. By examining which acceleration $A$ provides maximum crossambiguity value, the target bistatic acceleration can be estimated. The extended crossambiguity (6) can be calculated effectively by applying finite impulse response to a classical crossambiguity function (5). For details see \cite{18}, \cite{19}.

Let us analyze the influence of the integration time on acceleration resolution. The acceleration resolution can be calculated as:

$$\Delta A = \frac{\lambda}{T^2} \quad (7)$$

An example of acceleration resolution calculated according to (7) for 600 MHz is shown in Fig. 8. As can be seen, the dependency on $T^2$ is very strong. For integration times in the order of tens of milliseconds the acceleration resolution is very poor. Only when the integration time reaches values of several hundreds of milliseconds, the resolution reaches values in the order of 10 m/s$^2$. The acceleration resolution is connected to the acceleration estimation accuracy. In practice, the estimation accuracy can be much better than resolution – usually at least one order of magnitude.

The dependency of acceleration measurement accuracy on the integration time is illustrated in Fig. 9 on the measured echo of rocket. The plot shows the acceleration cross-section of the crossambiguity function (6), i.e. $\Psi_A(R_0, V_0, A)$, where $R_0, V_0$ are bistatic range and bistatic velocity of the target echo, respectively. For the shortest integration time (25 ms) the target amplitude is almost independent of the acceleration value assumed during calculations. For 50 ms a maximum appears indicating target bistatic acceleration in the range of 300-400 m/s$^2$. In the case of 100 ms integration time that would correspond to almost 3 dB of losses. For the longest integration time, equal to 200 ms, the acceleration profile does no longer have a sharp peak. This indicates that for such long integration time the acceleration changes, and the assumed signal model with target acceleration is no longer valid.

IV. CONCLUSION

In the paper selected aspects of rocket detection have been analyzed. The electromagnetic simulations of the RCS showed
that the rocket is reflecting strongly when illuminated from the boresight. As the rocket climbs, it is observed from the bottom, where the RCS is significantly lower (20 dB lower than the maximum in the analyzed case).

The analysis of the elevation patterns of the transmitter showed that capability of the radar to detect target can strongly depend on its altitude. This results form the fact that typically the transmitter antennas are designed to focus emitted power in a narrow beam pointed toward the earth surface. As a result, the airspace is illuminated to a large extent through the sidelobes of the elevation radiation pattern.

The measurement results confirmed expected behavior of the rocket – its acceleration is very high and can lead to echo spreading for higher values of integration times. This effect can be not only reduced, but in addition the acceleration of the elevation radiation pattern.

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