Multitarget Track-Before-Detect Based on Auxiliary Parallel Partition Particle PHD Filter

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Abstract—For dense multi-target scenarios, the existing Track-Before-Detect (TBD) algorithm based on Probability Hypothesis Density (PHD) has the shortcomings of underestimation of the number of targets and the waste of a large number of particles. The paper introduces the concept of two-layer particles and combines the Auxiliary Particle Filter (APF) PHD filter with the Parallel Partition (PP) theory to improve the estimation accuracy of the targets’ number and state. The simulation results have shown that, compared with the existing PF-PHD-TBD algorithm, the newly proposed algorithm has a significant improvement in the estimation of the targets’ number and the state, especially in dense multi-target scenarios.

Keywords—Parallel Partition; Auxiliary Particle Filter; Probability Hypothesis Density; Track-Before-Detect

I. INTRODUCTION

As for the detection and tracking problem in low SNR scenarios, the Track-Before-Detect (TBD) method is defined as a radar multi-frame signal accumulation technology, which directly uses the original form of observation signal, and utilizes the target motion information to assist detection. Due to the full use of the raw data, the TBD technology has been proved to improve the performance of the actual radar system. The Finite Set Statistics (FISST) method treats all of the targets’ state as a set-valued state, regards the observations obtained by one scan of radar as set-valued observations, and models the set-valued state and set-valued observations as Random Finite Set (RFS). The optimal Bayesian filter can be used to avoid the uncertainty of data association, so as to realize the multi-target estimation in the condition of detection uncertainty and clutter background. Maher R. approximates the multi-target posteriori density with Probability Hypothesis Density (PHD) [1]. At present, Gaussian Mixture PHD (GM-PHD) [2] and Sequential Monte-Carlo PHD (SMC-PHD) [3] are two convergence means of PHD which are widely used.

However, there are still many shortcomings in the field of PHD-TBD. In the multi-target detection and tracking problem, the dimensions of target state increases linearly according to the number of targets. The number of particles must be large enough to solve the high-dimensional problem [4]. In this paper, we introduce the concept of two-layer particle. Combining the theory of parallel partition and the auxiliary particle filter, this paper aims to improve the performance of SMC-PHD filter, and proposes an auxiliary parallel partition particle filter TBD algorithm based on PHD filter. When we predict and update an individual target’s state, the algorithm takes state likelihood of the targets which are adjacent to the individual into consideration, and then updates the state of this individual target of concern. Repeat this to the rest targets so that the target state can be accurately estimated when there are fewer particles or more targets. The simulation results have shown that the proposed algorithm has better target number and state estimation than PF-PHD-TBD, and this advantage is more obvious in the dense multi-target scenes.

II. TARGET MOTION MODEL AND OBSERVATION MODEL

A. Target motion model

We assume that $N_s$ is the number of targets at time $k$, the $i$th target’s state is $x^i_k$.

$$x^{i+1}_k = F_i x^i_k + w^i_k \quad t = 1, \cdots, N_s$$ (1)

where $x^i_k = [x^i_k, y^i_k, x_v^i, y_v^i, I^i_k]^T$ is the state vector of the $i$th target at time $k$. $I^i_k$ respectively represent the position, velocity and intensity information; $F_i = diag(F_x, F_y, 1)$ is the target state transition matrix, $F_x = F_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $w^i_k$ is the known white Gaussian process noise.

B. Radar observation model

The radar observation model aims to get the three-dimensional range-Doppler-bearing measurements of echo [5].

The measurement information $Z_k$ of one scan at time $k$ contains $N_s \times N_d \times N_b$ units, i.e.

$$Z_k = \sum_{i=1}^{N_s} \sum_{j=1}^{N_d} \sum_{l=1}^{N_b} z^i_{j,l}$$ (2)

where $z^i_{j,l}$ is the $i$th range unit, the $j$th Doppler unit, the $l$th bearing unit echo information at time $k$ [6].

$$z^i_{j,l} = [s^A_{i,j,l} s^B_{i,j,l}]$$ (3)
\( z_{a, j, l}^{k, j} \) denotes the target complex amplitude information.

\[
\begin{align*}
&z_{a, j, l}^{k, j} = \\
&\begin{cases} \\
A \sum_{r = 1}^{N} h_{r, r, l}^{k, j}(x_r^j) + v_{r, r, l}^{k, j}, H_l & \text{if } \text{target exists} \\
0 & \text{if } \text{no target exists}
\end{cases}
\end{align*}
\]

where \( H_l \) indicates that there is a target, while \( H_0 \) indicates no target exists; \( A \) denoting the complex echo of target \( x \), which for a Swerling 0 model is constant in modulus; measurement noise \( v_{r, r, l}^{k, j} \) is additive complex white Gaussian noise, of which the cophase component and quadrature component are of the same Gaussian distribution (variances are \( \sigma^2 \)).

\( \lambda_{a, j, l}^{k, j}(x_r^j) \) is the point spread function which is defined by

\[
\lambda_{a, j, l}^{k, j}(x_r^j) = \exp\left\{-\frac{(r_i - r_i^j)^2}{2R} \lambda_i - \frac{(d_j - d_j^i)^2}{2D} \lambda_d \right\} - \frac{(b_i - b_i^j)^2}{2B} \lambda_b
\]

where \( R, D \) and \( B \) denote the range, Doppler and bearing unit's resolution respectively, and they are decided by the bandwidth, the accumulation time and the radar beam width; \( L_r, L_d \) and \( L_b \) respectively denote the loss factors in the three dimensions; \( r_i^j \), \( d_i^j \) and \( b_i^j \) are range, Doppler and bearing units where the \( r \)th target is located at time \( k \) [7].

III. EXISTING PF-PHDF-TBD ALGORITHM

There are many similarities between the PF-PHDF-TBD and the PF-PHDF filter, except that the former needs to detect and estimate the target in the case of unknown target number and state distribution uncertainty. What’s more, the PHD update processes of both algorithms are also different [8].

A. Initialization

We sample a certain number of particles according to the target’s initial recommended density.

\[
x_r^0 = p_0(X^0)
\]

B. PHD prediction

Assuming that there are \( L_k \) particles at time \( k \), the particles of the surviving targets are sampled based on the suggested density \( q_k (\cdot | x_r^k, Z^{k+1}) : \)

\[
x_r^{k+1} = q_k (\cdot | x_r^k, Z^{k+1}). i = 1, 2, \ldots, L_k
\]

In addition, assuming that there are \( J_{k+1} \) particles of the newborn targets at time \( k+I \), the particles of the newborn targets are sampled according to the suggested density \( p_{k+1} (\cdot | Z_{k+1}) : \)

\[
x_r^{k+1} = p_{k+1} (\cdot | Z_{k+1}). i = L_k + 1, \ldots, L_k + J_k
\]

Then calculate the weights of surviving and newborn particles according to the following:

\[
\alpha_{k+1}^{i} = \frac{\alpha_{k}^{i} f_{X_{k+1}}^{i} (x_r^{i+1}) p_{k+1} (x_r^{i+1} | x_r^{i}, Z^{k+1})}{\sum_{i = 1}^{L_k + J_k} \alpha_{k}^{i} f_{X_{k+1}}^{i} (x_r^{i+1}) p_{k+1} (x_r^{i+1} | x_r^{i}, Z^{k+1})}
\]

\[
\alpha_{k+1}^{i} = \frac{\alpha_{k}^{i} (x_r^{i+1} | x_r^{i}, Z^{k+1})}{\sum_{i = 1}^{L_k + J_k} \alpha_{k}^{i} (x_r^{i+1} | x_r^{i}, Z^{k+1})}
\]

where \( \lambda_{k+1} \) is the clutter and noise constant; \( g(z_{i,j}^{k+1} | x_r^{i+1}) \) refers to observation likelihood function. While the other one in the denominator \( \lambda_{k+1}^{i} (z_{i,j}^{k+1}) \) is calculated as follow.

\[
\rho_{k+1} (z_{k+1} | X^{k+1}) = \\
\sum x_r^{k+1} \alpha_{k+1}^{i} (X_r^{k+1}) \alpha_{k+1}^{i} (X_r^{k+1})
\]

\[
P_{X_{k+1}} = \left\{ x_r^{k+1} : p \in \{ 1, 2, \ldots, L_k + J_k \} \right\}
\]

\[
\tilde{p} = D (x_r^{k+1}), \tilde{p} = D (x_r^{k+1}), \tilde{p} = D (x_r^{k+1})
\]

D. Resampling and state extraction

The integral of the PHD in the measurement area is the number of targets we get by estimation. In the case of particle filter, the number of targets is estimated as the sum of the weights of the particles after resampling.

\[
\tilde{M} (k + 1) = \left[ M (k + 1) \right]_{\text{sum}} = \sum_{i = 1}^{L_k + J_k} \alpha_{k+1}^{i}
\]

It should be pointed out that the PHD filter needs to satisfy two conditions for the approximation of the multi-target Bayesian filter [9]: firstly, the number of false alarms per frame follows a Poisson distribution; secondly, the multi-target observation model must be a standard form, i.e.
\[
\mathbf{g}_i \left( \mathbf{z}_{ij}^t | \mathbf{x}_i^t \right) = \frac{1}{2\pi \sigma^2} \exp \left\{ -\frac{\mathbf{z}_{ij}^t \mathbf{\mu}_i^t \mathbf{z}_{ij}^t}{2\sigma^2} \right\} \times I_0 \left( \frac{\sqrt{\mathbf{z}_{ij}^t}}{\sigma} \right) \tag{14} 
\]

\[ \mathbf{\mu}_i^t = A_i \mathbf{b}_i^{k+l} \left( \mathbf{x}_i^t \right) \tag{15} \]

\[ I_0 \] is the zero-order Bessel function. PF-PHD-TBD needs to rely on a sufficient number of particles. In the multi-target detection and tracking problem, the dimensions of the target state increase linearly when the number of targets increases. To achieve the demanded filtering effect, the number of particles must be large enough to solve the high-dimensional problem. In order to solve the contradiction between computational complexity and estimation accuracy, this paper applies a new thought called the auxiliary parallel partition PF-PHD-TBD.

IV. APP-PF-PHD-TBD ALGORITHM

A. Particle parallel partition theory

Posterior independence assumption [10] assumes that the posterior probabilities of the targets are independent of each other. So the influence of the curse of dimensionality can be alleviated in the prior probability estimate of the next filter iteration. This is validated by the examples of independent partition (IP) and parallel partition (PP) [11].

Suppose that the total number of targets is known and the motions of them are independent at time \( t \), the \( i \)th particle \( \mathbf{x}_i^t \) can be defined as \( \mathbf{x}_i^t = \left[ \left( \mathbf{x}_{1i}^t \right)^T, \ldots, \left( \mathbf{x}_{Ni}^t \right)^T \right]^T \), where \( \mathbf{x}_{ji}^t \), \( t = 1,2,\ldots,N_i \) refers to the second layer particles that corresponds to the target.

According to the posterior independent assumption, the posterior probability density can be rewritten as [12]:

\[
p \left( \mathbf{x}_{i}^{k+1} | \mathbf{z}^{k+1} \right) \propto p \left( \mathbf{z}^{k+1} | \mathbf{x}_i^{k+1} \right) \prod_{j=1}^{N_i} \sum_{i=1}^{N_i} w_i^j \ p \left( \mathbf{x}_i^{k+1} | \mathbf{x}_j^{k+1} \right) \tag{16} 
\]

In order to consider the impact of the adjacent objects in the sampling step, the average predicted target state for the target is

\[
\hat{\mathbf{x}}_i^{k+1} = \sum_{i=1}^{N_i} w_i^j \cdot \mathbf{x}_i^{k+1|k} \tag{17} 
\]

where \( \mathbf{x}_i^{k+1|k} \) is the prediction of the \( i \)th particle at time \( k+1 \) [13]. In addition, another set of vectors \( \hat{\mathbf{x}}_i^{k+1|k} \) represents a set of predicted states for each target (except for the \( i \)th target) at time \( k+1 \).

\[
\hat{\mathbf{x}}_i^{k+1|k} = \left[ \left( \hat{\mathbf{x}}_i^{k+1} \right)^T, \ldots, \left( \hat{\mathbf{x}}_{1i}^{k+1} \right)^T, \left( \hat{\mathbf{x}}_{2i}^{k+1} \right)^T, \ldots, \left( \hat{\mathbf{x}}_{Ni}^{k+1} \right)^T \right] \tag{18} 
\]

Then the target predict likelihood is defined as \( b_t \left( \mathbf{x}_i^{k+1} \right) \propto p \left( \mathbf{z}^{k+1} | \hat{\mathbf{x}}_i^{k+1|k} \right) \cdot \mathbf{x}_i^{k+1|k} \). However, the problem of parallel partition is that its description about the dense targets is rely on the accurate sampling of the estimated state. Such a partition tends to be inaccurate or misleading because the accurate sampling is not always accomplished. A feasible solution is to combine the partition with auxiliary particle filter.

B. Auxiliary parallel partition-particle filter (APP-PF)

Auxiliary particle filter is designed as a better approach to measure and simulate the optimal importance sampling process by means of auxiliary variables. However, when the number of targets increases, the sampling process will be impacted due to the curse of dimensionality in jointly sampling the entire state space. Then APF’s performance will be degraded. APP-PF is a method that combines particle parallel partition and auxiliary particle filter to realize a better performance.

The auxiliary variable \( a \) of APP-PF is derived from a distribution of \( \lambda_{ij} \). The distribution jointly defined by the prediction target likelihood \( b_t \left( \mathbf{x}_i^{k+1} \right) \) and the particle weights.

\[
\lambda_{ij} = b_t \left( \mathbf{\mu}_{i}^{k+1} \right) w_i^j \tag{19} 
\]

\[
a = \left[ a_1, a_2, \ldots, a_{N_i} \right]^T \] is the result of sampling from the distribution defined by \( \left( \lambda_{1}, \lambda_{2}, \ldots, \lambda_{N_i} \right) \).

For APP-PF, the importance density functions are independent in target sampling and they are expressed as following.

\[
q \left( \mathbf{x}_i^{k+1}, a | \mathbf{z}^{k+1} \right) = \prod_{i=1}^{N_i} q_i \left( \mathbf{x}_i^{k+1}, a_i | \mathbf{z}_i^{k+1} \right) \tag{20} 
\]

\[
q_i \left( \mathbf{x}_i^{k+1}, a_i | \mathbf{z}_i^{k+1} \right) = b_t \left( \mathbf{\mu}_{i}^{k+1} \right) w_i^j p \left( \mathbf{x}_i^{k+1} | \mathbf{x}_i^{k+1} \right) \tag{21} 
\]

where \( \mathbf{\mu}_{i}^{k+1} \) corresponds to the second layer particle \( \mathbf{x}_i^{k+1} \) and represents a sample of \( \mathbf{x}_i^{k+1} \), i.e. \( \mathbf{\mu}_{i}^{k+1} \sim p \left( \mathbf{x}_i^{k+1} | \mathbf{x}_i^{k+1} \right) \); \( w_i^j \) denotes the weight of \( \mathbf{x}_i^{k+1} \).

After sampling the auxiliary variables, equation (15) can be rewritten as the following.

\[
p \left( \mathbf{x}_i^{k+1}, a | \mathbf{z}^{k+1} \right) \propto p \left( \mathbf{z}^{k+1} | \mathbf{x}_i^{k+1} \right) \prod_{i=1}^{N_i} w_i^j p \left( \mathbf{x}_i^{k+1} | \mathbf{x}_i^{k+1} \right) \tag{22} 
\]

Because of the auxiliary variable \( a \), the computational complexity of (21) is reduced when compared with the one of (15). The second layer particles (also called sub-particles) together affect the formation of particles in the next time step. In particular, the formation of particles is based on the selection of sub-particles corresponding to high-likelihood ratio particles in the next time step. In addition, it should be noted that for tracking the single target, i.e. \( N_i = 1 \), APP-PF is the standard auxiliary particle filter.

C. APP-PF-PHD-TBD algorithm

The difference between the APP-PF-PHD-TBD and PF-PHD-TBD lies in the prediction and updating process.
• prediction

Supposing no new target derives from existing targets, we are given the measurement at $z^{k+1}$ time $k+1$ and a set of weighted particle $\{x_{i}^{1}, w_{i}^{1}, (x_{i}^{2}, w_{i}^{2}) \cdots (x_{i}^{N}, w_{i}^{N})\}$.

With the help of the $ith$ sub-particle of particle $i$, we use the prediction probability $p(x_{i}^{k+1} | x_{i}^{k})$ of the particle to obtain $\mu_{i}^{k+1}$, calculate the predicted target likelihood $b_{i}^{k+1}(\mu_{i}^{k+1})$, obtain $\lambda_{i}^{k+1}$, normalize $(\lambda_{i+1, 1}, \lambda_{i+1, 2}, \cdots, \lambda_{i+1, N})$, and obtain the auxiliary variable $a_{i}^{k+1}$ from the distribution of $\lambda_{i+1, i}$.

The predicted weights of the sub-particles are obtained according to the normal PF-PHD-TBD prediction, i.e.,

$$o_{i}^{k+1} = \frac{\lambda_{i+1, i}^{|Z_{k+1}|} |x_{i}^{k+1}|}{\sum_{i}^{N} \lambda_{i+1, i}^{|Z_{k+1}|} |x_{i}^{k+1}|} \cdot P_{k+1}(x_{i}^{k+1} | Z_{k+1})$$

(23)

Then the prediction probability $p(x_{i}^{k+1} | x_{i}^{k})$, which is defined by the auxiliary variable, to sample sub-particles at time $k+1$.

- updating

Substituting equation (19) and (20) into the particle weight update equation (10), we can update the particles’ weights.

$$o_{i}^{k+1} = \frac{\prod_{j \in I} b_{i}(\mu_{i}^{(j)})}{\prod_{j \in I} \lambda_{i+1, j} |x_{i}^{k+1}|} \cdot o_{i}^{k+1}$$

(24)

In order to complete the updating above, we need to update the weights of the first layer particles $w_{i}^{k+1}, i = 1, \cdots, N$, i.e.,

$$w_{i}^{k+1} \propto \frac{p(x_{i}^{k+1} | x_{i}^{k+1})}{\prod_{i=1}^{N} b_{i}(\mu_{i}^{k+1})} \cdot \prod_{i=1}^{N} w_{i}^{k+1}$$

$$\propto \frac{1}{\prod_{i=1}^{N} b_{i}(\mu_{i}^{k+1})} \cdot \prod_{i=1}^{N} w_{i}^{k+1}$$

(25)

Although the form is similar to what is described in Part II, the difference is that the particles are already the result of combining sub-particles’ weights.

V. SIMULATION RESULT ANALYSIS

A. Simulation conditions

This section presents the examples of multi-target TBD scenarios. It was assumed that the target’ motion satisfies the CV (Constant Velocity) model.

Sampling period $T = 1s$; the targets’ tracks are approximately linear; the radar surveillance area is set to $[0, 2000m] \times [0, 2000m]$; the range resolution is set as 10m; the Doppler resolution is set 1m/s; the beam resolution is set as 1°. For the sake of the comparison of algorithm performance, the simulation generated 40 frames of radar data, and took 100 Monte Carlo experiments to analyze the mean value. In order to demonstrate the applicability of the proposed algorithm to dense target scene, this paper uses two scenarios. The initial state 1 corresponds to discrete targets scenario and the initial state 2 corresponds to dense targets scenario where the trajectories repeatedly cross each other. Both of the scenarios are shown in Table I.

<table>
<thead>
<tr>
<th>Target state</th>
<th>initial state 1</th>
<th>initial state 2</th>
<th>Appear frame</th>
<th>Disappear frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial state 1</td>
<td>[50,55,750,0,1]</td>
<td>[50,55,750,0,1]</td>
<td>5</td>
<td>36</td>
</tr>
<tr>
<td>initial state 2</td>
<td>[50,75,400,0,1]</td>
<td>[1600,75,400,25,1]</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>initial state 3</td>
<td>[50,100,1250,0,1]</td>
<td>[500,50,1250,50,1]</td>
<td>16</td>
<td>33</td>
</tr>
<tr>
<td>initial state 4</td>
<td>[1250,45,1500,25,1]</td>
<td>[150,75,1250,80,1]</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>initial state 5</td>
<td>[50,60,1900,0,5,1]</td>
<td>[150,0,1000,60,1]</td>
<td>15</td>
<td>31</td>
</tr>
<tr>
<td>initial state 6</td>
<td>[500,90,1000,0,2,3]</td>
<td>[500,0,6,600,50,1]</td>
<td>17</td>
<td>30</td>
</tr>
</tbody>
</table>

The target survival probability $P_{s} = 0.99$, the birth probability $p_{b} = 0.01$; the power spectral density of process noise is 0.001; the initial particle number $L = 500$ or 300; the signal-to-noise ratio SNR = $10 \log (P / 2 \sigma^{2})$, let $\sigma^{2} = 1$ and we can derive the complex amplitude $A_{k}$; The targets’ intensities was set as 20. The entire simulation scenario does not consider about any one new target derives from existing targets.

Assume that the location of clutter points is uniformly distributed in the detection area at each time, and its number obeys a poisson distribution with an average value of 20. Under the condition of a clutter density of $\lambda_{c} = 10^{-4} m^{-2}$.

B. Simulation results and analysis

The OSPA is used as the evaluation criterion of the target state estimation accuracy. Suppose there are two sets $X$ and $\hat{X}$. The OSPA distance contains two parts called the location error $OSPA_{loc}$ and the cardinality error $OSPA_{card}$. The OSPA distance parameter $c = 10 m$ and $p = 2$ is the distance parameter. Simulation were established when SNR=9dB, 8dB, 6dB.

First, we analyzed the number estimation discretely, which was shown in Fig.1. APP-PF-PHD-TBD algorithm has a similar
performance of target number estimation compared with PF-PHD-TBD algorithm when it comes to the discrete targets scenario. From Fig. 2, the APP-PF-PHD-TBD algorithm has significantly better performance than the PF-PHD-TBD algorithm in dense-targets estimation.

From Fig. 2 and Fig. 3, we have the result that the performance of APP-PF-PHD-TBD is still better than that of traditional PHD-TBD although the low signal-to-noise ratio and small number of particles.

Through the comparison, it can be concluded that the proposed algorithm is more credible in the cases of low SNR, which verifies the superiority of the parallel partition theory. But the computational efficiency of the APP-PF-PHD-TBD is slightly lower due to the two-layer particle concept.

VI. CONCLUSION

Combining with particle parallel partition algorithm, this paper was based on the sequential Monte-Carlo probability hypothesis density filter and applied to the TBD scenario. We mainly focus on the target detection problem in dense targets scenario when the number of particles is small and the SNR is low. Then we illustrate the system model, observation model, and the main process of the algorithm we proposed. The simulation results showed that APP-PF-PHD-TBD algorithm is superior in dense targets scenario and it has a better target estimation under the same simulation conditions. But the disadvantage is clear that the computational burden is inevitably increased and the efficiency is lower.

REFERENCES


Fig. 1. Two algorithm’s estimation of target’s number under discrete targets scene and different numbers of particles

(a) States estimation accuracy comparison at SNR=9dB
(b) States estimation accuracy comparison at SNR=8dB
(c) States estimation accuracy comparison at SNR=6dB

Fig. 2. Two algorithm’s estimation accuracy of targets’ number under dense targets scene and different numbers of particles

(a) States estimation accuracy comparison at SNR=9dB
(b) States estimation accuracy comparison at SNR=8dB
(c) States estimation accuracy comparison at SNR=6dB

Fig. 3. Two algorithm’s estimation accuracy of targets’ state under dense targets scene and different numbers of particles

(a) Covered area
(b) Coverage error
(c) Detection rate
(d) False alarm rate