High-Order Phase Correction for Ground Moving Target Imaging in High-Squint SAR Mode

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Abstract—For desired resolution, high-squint SAR has a long-time coherent processing interval (CPI). In this case, the maneuvering motion of moving target can not be neglected, which causes high-order phase terms in the echoed data. Many ground moving target imaging (GMTIm) algorithms have been proposed for broadside mode, but they are not suitable for high-squint mode. In the proposed GMTIm method, we assume that the target has constant velocity in subaperture CPI, but maneuvering motion parameters for the whole CPI. Within the subaperture CPI, the target signal can be simplified as a three-order phase function, and the instantaneous Doppler frequency (DF) is estimated by some time-frequency analysis tools. For the whole CPI, the subaperture instantaneous DF is combined to form a least square problem, outputting the undetermined phase coefficients. The proposed GMTIm method can estimate and correct more high-order phase terms in comparison with the existing methods. The effectiveness of the proposed method is demonstrated by real-measured high-squint SAR data.

Keywords—Ground moving target imaging (GMTIm), high-order phase correction, high-squint SAR, maneuvering motion, parameter estimation.

I. INTRODUCTION

For synthetic aperture radar (SAR), the high-squint imaging mode enhances the flexibility and detection ability of the modern radar system, which makes it more preferable than the conventional broadside mode [1]-[3]. In high-squint mode, focusing ground moving target can provide detailed target signatures, which benefits the subsequent detection and classification.

In stationary SAR image, the ground moving target is defocused and dislocated due to their unknown motions. Generally, the range velocity induces an extra Doppler centroid shift, which causes azimuth dislocation and additional linear range walk. The cross-range velocity impacts the Doppler rate, which results in cross-range defocusing. In addition, the high-squint SAR has a large coherent processing interval (CPI), where the maneuvering motion of the observed target is usually inevitable. The maneuvering motion parameters, such as acceleration and acceleration rate, cause high-order phase terms. To achieve accurate focusing, these high-order phase terms should be estimated and corrected.

Moving target detection [4]-[6], [12] is a necessary step before focusing. For the target located at the high-band of the pulse repetition frequency (PRF), it can be separated from the clutter spectrum in range-Doppler domain. More details of moving target detection will be presented in another paper.

Many GMTIm algorithms have been proposed in literatures [4]-[12]. Keystone transform and Hough transform (HT) can be used to correct the linear range walk. If range curve exists for the target of interest, second-order keystone transform can be utilized. After range migration correction, target energy is concentrated in one range cell (the case of extending target will be considered in future work). For the signal in this range cell, fractional Fourier transform (FrFT) is capable of extracting Doppler rate, and the third-order Doppler parameter can be estimated by generalized Hough-high-order ambiguity function (GHHAF) transform or polynomial Fourier transform. For our knowledge, the existing GMTIm algorithms are suitable for broadside imaging. They can not be applied directly to high-squint mode, since more high-order phase terms should be considered. It is worth mentioning that high-order phase terms are not only caused by SAR observation, but also induced by target maneuvering motion in a large CPI.

II. MOVING TARGET SIGNAL MODEL IN HIGH-SQUINT SAR

Fig. 1 shows the high-squint SAR geometry containing one moving target. The platform carrying SAR sensor flies along the straight line parallel to X-axis with a constant velocity v. The squint angle of the beam center line is θ, and the operational range from aperture center to scene center o is r0. Suppose that a target with azimuth distance xT moves from position P1 to P2 during the CPI. It is hard to determine the target motion information, since no prior knowledge is available. For example, the moving target may have constant velocity, acceleration, or time-varying motion parameters. For
general, the along-track motion is expressed by vector $\Lambda_x$, and the cross-track one is defined as $\Lambda_y$. Both $\Lambda_x$ and $\Lambda_y$ contain velocity, acceleration and high-order motion parameters. As a result, the slant range of the moving target at slow time $t_a$ can be expressed by

$$ r_T(t_a) = \sqrt{[r_0 \sin \theta - v(t_a - t_c) + s(\Lambda_x)]^2 + [r_0 \cos \theta - s(\Lambda_y)]^2} $$

(1)

where $t_a \in (-T/2, T/2)$. $T$ is the synthetic aperture time, $t_c$ denotes the aperture center time for moving target, $s(\Lambda_x)$ and $s(\Lambda_y)$ represent the moving distance in $X$- and $Y$-axis, respectively. Extending (1) into $Q$-order Taylor series at $(t_a - t_c)$, we get the approximated slant range

$$ r_T(t_a) \approx \sum_{q=0}^{Q} a_q(t_a - t_c)^q $$

$$ \approx \sum_{q=0}^{Q} \alpha_q t_a^q $$

(2)

where $a_q$ is the $q$th Taylor coefficient. $\alpha_q$ is a function of $a_q$ and $t_c$, which means that $\alpha_q$ is azimuth dependent.

For stationary scene imaging in high-squint mode, five-order polynomial has been used in some literatures. In the presence of maneuvering motion, more high-order polynomial coefficients should be considered for accurate focusing. The proposed GMTIm method aims to estimate all $Q$ ($a_0$ is omitted since it has no effect on focusing) unknown coefficients using a local-to-global processing strategy.

III. THE PROPOSED GROUND MOVING TARGET IMAGING METHOD

In a short subaperture CPI, the maneuvering motion of target can be neglected. Therefore, we assume that the moving target has constant velocity within subaperture CPI, and the slant range at subaperture slow time $t_{a_{sub}}$ can be simplified as

$$ r_T(t_{a_{sub}}) = \sqrt{[r_0 \sin \theta - v(t_{a_{sub}} - t_c) + v_x(t_{a_{sub}} - t_c)]^2 + [r_0 \cos \theta - v_y(t_{a_{sub}} - t_c)]^2} $$

(3)

where $t_c = x_T/(v - v_x)$, $t_{a_{sub}} \in (-T_{sub}/2, T_{sub}/2]$, $T_{sub}$ is the subaperture duration, $v_x$ and $v_y$ are the along-track and cross-track velocity of target, respectively. From our experience, three-order Taylor series of (3) is accurate enough to focus subaperture signal, which is given by

$$ r_T(t_{a_{sub}}) \approx \sum_{k=0}^{3} c_k (t_{a_{sub}} - t_c)^k $$

(4)

where

$$ \begin{cases} 
    c_0 = r_0 \\
    c_1 = -v \sin \theta + v_x \\
    c_2 = \frac{(v \cos \theta - v_x)^2}{2r_0} \\
    c_3 = \frac{(v \cos \theta - v_x)^2(v \sin \theta - v_y)}{2r_0^2}
\end{cases} $$

(5)

and

$$ \begin{cases} 
    v_r = v_x \sin \theta - v_y \cos \theta \\
    v_{cr} = v_x \cos \theta + v_y \sin \theta
\end{cases} $$

(6)

$v_r$ is the range velocity, and $v_{cr}$ is the cross-range one.

Suppose that a pulsed chirp signal $s(t_r) = rect(t_r/T_p) \cdot \exp \left[ j2\pi \left( f_c t_r + \gamma t_r^2 / 2 \right) \right]$ is transmitted. Symbol $t_r$ is the fast time, $T_p$ denotes the pulse width, $f_c$ represents the carrier frequency, and $\gamma$ corresponds to the chirp rate. After removing the carrier frequency, matched-filtering in range dimension, and transforming the echoed signal into range-frequency domain, we get

$$ S_s(f_r, t_{a_{sub}}) = W_r(f_r) w_a(t_{a_{sub}} - t_c) \cdot \exp \left[ -\frac{4\pi}{c} (f_c + f_r) \sum_{k=0}^{3} c_k (t_{a_{sub}} - t_c)^k \right] $$

$$ = W_r(f_r) w_a(t_{a_{sub}} - t_c) \cdot \exp \left[ -\frac{4\pi}{c} (f_c + f_r) \sum_{k=0}^{3} \beta_k t_{a_{sub}}^k \right] $$

(7)

where $W_r(f_r)$ and $w_a(t_{a_{sub}} - t_c)$ are the window function in range and cross-range dimensions, respectively, $c$ denotes the speed of light, and $f_c$ represents the range frequency. The coefficient $\beta_k$ is given by

$$ \begin{cases} 
    \beta_0 = r_0 - c_1 t_c + c_2 t_c^2 - c_3 t_c^3 \\
    \beta_1 = c_1 - 2c_2 t_c + 3c_3 t_c^2 \\
    \beta_2 = c_2 - 3c_3 t_c \\
    \beta_3 = c_3
\end{cases} $$

(8)

Generally, $\beta_3$ makes no contribution to the range cell migration, and the range curve can be corrected by SAR parameters. Target motion mainly affects the linear range walk and azimuth phase. More details about these approximations will be presented in another paper. The range curve correction function is given by

$$ F_{RCC}(f_r, t_{a_{sub}}) = \exp \left[ \frac{2\pi (v \cos \theta)^2}{c r_0} f_r t_{a_{sub}} \right] $$

(9)

After range curve correction, the signal is transferred back into the fast time domain, which is given by

$$ ss(t_r, t_{a_{sub}}) = \text{sinc} \left\{ \gamma T_p \left[ t_r - \frac{2(\beta_0 + \beta_1 t_{a_{sub}})}{c} \right] \right\} \cdot \exp \left[ -\frac{4\pi}{\lambda} \sum_{k=0}^{3} \beta_k t_{a_{sub}}^k \right] $$

(10)

where $\lambda$ is the wavelength. Obviously, $\beta_1 t_{a_{sub}}$ introduces extra linear range walk in (10). We use HT to estimate the target envelop slope rate. Then the linear range walk correction function can be obtained as follows

$$ F_{LRWC}(f_r, t_{a_{sub}}) = \exp \left[ \frac{4\pi}{c} (f_c + f_r) \hat{\beta}_1 t_{a_{sub}} \right] $$

(11)

where $\hat{\beta}_1$ is the target envelop slope rate estimated by HT.
After linear range walk correction, the target signal is given by

\[
ss(t_r, t_{a_{sub}}) = \text{sinc} \left[ \gamma T_p (t_r - \frac{2\beta_0}{c}) \right] 
\cdot \exp \left[ -j \frac{4\pi}{\lambda} (\beta_0 + \beta_2 t_{a_{sub}}^2 + \beta_3 t_{a_{sub}}^3) \right]
\] (12)

Now the target energy has been concentrated into one range cell. One can see that the target locates at range \( \beta_0 \) instead of \( r_0 \) after linear range walk correction, which results in azimuth-dependent phase. The azimuth-dependence of phase in (12) will be analyzed in detail in another paper.

Next, the range cell containing target signal is extracted, and FrFT is utilized to estimate the Doppler rate. Then we can get the second-order deramping function as follows

\[
F_{2nd_{\text{dmp}}}(t_{a_{sub}}) = \exp \left( j \frac{4\pi}{\lambda} \beta_2 t_{a_{sub}}^2 \right)
\] (13)

where \( \beta_2 \) comes from the estimation result of FrFT. Multiplying (12) by (13), the target signal can be rewritten as

\[
ss(t_r, t_{a_{sub}}) = \text{sinc} \left[ \gamma T_p (t_r - \frac{2\beta_0}{c}) \right] 
\cdot \exp \left[ -j \frac{4\pi}{\lambda} (\beta_0 + \beta_3 t_{a_{sub}}^3) \right]
\] (14)

To estimate the third-order phase in (14), we design an iterative searching method. From the output of HT and FrFT, we can get the coarse values of \( v_r \) and \( v_{cr} \). In (8), \( \beta_1 \) is dominated by azimuth-independent coefficient \( c_1 \), and the azimuth-dependent one \((-2c_2 t_c + 3c_3 t_c^2)\) is relative small. Then a coarse value for \( v_r \) can be calculated by \( \hat{v}_r = \beta_1 + v \sin \theta \). Similarly, a coarse value of \( v_{cr} \) is estimated by \( \hat{v}_{cr} = v \cos \theta - \sqrt{2r_0\beta_2} \).

Suppose the true velocity \( v_r \in [\hat{v}_r - \Delta v/2, \hat{v}_r + \Delta v/2] \) and \( v_{cr} \in [\hat{v}_{cr} - \Delta v'/2, \hat{v}_{cr} + \Delta v'/2] \) (\( \Delta v > 0 \)), we can get a searching scope \( \beta_3 \in [\beta_{3_{\text{min}}}, \beta_{3_{\text{max}}}] \). The minimum and maximum values of \( \beta_3 \) are given by

\[
\beta_{3_{\text{min}}} = \frac{[v \cos \theta - (\hat{v}_{cr} + \Delta v'/2)]^2 [v \sin \theta - (\hat{v}_r + \Delta v/2)]}{2\hat{\beta}_2^2}
\] (15a)

\[
\beta_{3_{\text{max}}} = \frac{[v \cos \theta - (\hat{v}_{cr} - \Delta v'/2)]^2 [v \sin \theta - (\hat{v}_r - \Delta v/2)]}{2\hat{\beta}_2^2}
\] (15b)

The third-order deramping function is set as

\[
F_{3rd_{\text{dmp}}}(t_{a_{sub}}) = \exp \left( j \frac{4\pi}{\lambda} \beta_3 t_{a_{sub}}^3 \right)
\] (16)

where \( \beta_3 \in [\beta_{3_{\text{min}}}, \beta_{3_{\text{max}}}] \). Multiplying (14) by (16) and performing azimuth Fourier transform (FT), the target Doppler profile is generated. Based on the criterion of minimum entropy, the optimal estimator of \( \beta_3 \) can be found. At the same time, the target can be focused in Doppler domain as follows

\[
ss(t_r, f_{a_{sub}}) = \text{sinc} \left[ \gamma T_p (t_r - \frac{2\beta_0}{c}) \right] 
\cdot \text{sinc} \left( \frac{f_{a_{sub}}}{B_{a_{sub}}} \right) \cdot \exp \left( -j \frac{4\pi}{\lambda} \beta_0 \right)
\] (17)

where \( B_{a_{sub}} \) is the Doppler bandwidth in subaperture CPI.

Based on the estimated \( \beta_1, \beta_2 \) and \( \beta_3 \), we obtain the instantaneous DF for each subaperture CPI as follows

\[
DF(m; t_{a_{sub}}) = \frac{2}{\lambda} \left[ \beta_1(m) + 2\beta_2(m) t_{a_{sub}} + 3\beta_3(m) t_{a_{sub}}^2 \right]
\] (18)

where \( m \) is the index of subaperture, \( m = 1, 2, \ldots, M \). The DF elements of all subapertures can be vectorized as

\[
\Psi = \begin{bmatrix} DF(1; t_{a_{sub}}) \\ DF(2; t_{a_{sub}}) \\ \vdots \\ DF(M; t_{a_{sub}}) \end{bmatrix}_{N \times 1}
\] (19)

where \( N \) represents the number of pulses.

Multiplying the first-order derivative of the slant range in (2) by \( 2/\lambda \), the instantaneous DF for the whole CPI is expressed by the following matrix

\[
\Omega = \frac{2}{\lambda} \cdot \frac{\partial r_T(t_a)}{\partial t_a} = \frac{2}{\lambda} \cdot [\delta(n, q)]_{N \times Q}
\] (20)

\[
\delta(n, q) = q \cdot [t_a(n)]_n^{-1}
\] (21)

where \( n = 1, 2, \ldots, N \), and \( q = 1, 2, \ldots, Q \).

The unknown coefficients in (2) can be written into a vector form \( \alpha = [\alpha_1 \alpha_2 \cdots \alpha_Q]^T \), where \([\cdot]^T\) denotes the vector/matrix transpose. Then we get the following equation

\[
\Psi = \Omega \alpha
\] (22)

Fig. 2. Flowchart of the proposed GMTIm method.
The least square estimation for $\alpha$ is now available

$$\hat{\alpha} = (\Omega^T \Omega)^{-1} \Omega^T \Psi$$

(23)

where $(\cdot)^{-1}$ is the matrix inverse.

According to the estimator $\hat{\alpha}$, the range cell migration and azimuth phase can be corrected for the whole CPI data, then the target can be focused in the range-Doppler domain. Since the number of DF $N$ is much larger than the order of polynomial $Q$, the proposed GMTIm algorithm is suitable for high-order signal model. For clarity, the algorithm procedure of the proposed GMTIm method is presented in Fig. 2. In practical application, the proposed GMTIm may be performed in an iterative way to improve the focusing precision. In earlier iterations, a short subaperture CPI is recommended for high-order parameter estimation, while a long subaperture CPI is suitable for low-order signal model in later iterations.

IV. EXPERIMENTAL RESULTS

The experimental SAR system works at Ku band and the operational range from aperture center to scene center is 7.5km. The recorded dataset has a squint angle high up to $70^\circ$, and the corresponding CPI is 4.096s. The resolution of the generated SAR image is about $2m \times 2m$ (range $\times$ cross-range). Two moving targets are presented in the following, where the first target locates at the high-band of PRF, and the second one is submerged by the ground clutter.

Case 1: After linear range walk correction by SAR processing, the echoed data is transferred into the range-Doppler domain, as shown in Fig. 3(a). Obviously, a moving target can be indicated in the high-band of PRF, which means that the target has a large range velocity. The target detection result is given in Fig. 3(b). Fig. 3(c) shows the stationary clutter image. One can see that this scene contains a highway, thus moving target appears easily in SAR data.

The data in Fig. 3(b) is transferred back into slow time domain by azimuth inverse FT, generating the target trajectory in Fig. 4(a). In the proposed GMTIm method, the whole CPI is divided into 4 subapertures, and the moving target signal model is set to be a 7-order polynomial. After correction, the target energy is concentrated into one range cell, as shown in Fig. 4(b). Applying azimuth FT, the moving target can be focused, as illustrated in Fig. 5(a). For comparison, the result of the GMTIm method in [7] is presented in Fig. 5(b), which involves obvious defocusing effect. The referencing method in [7] only considers 3-order phase correction, which is not accurate enough to focus this moving target.

Case 2: The imaging scene containing the second moving target is shown in Fig. 6(a). There is a road along the cross-range dimension, which means that the moving target involves dominant cross-range motion parameters. After target detection in the image domain, the moving target energy is presented in Fig. 6(b).

In the proposed GMTIm method, the whole CPI is divided
into 4 subapertures, and the moving target signal model is set to be a 7-order polynomial. Fig. 7 gives the target trajectory, where Fig. 7(a) is the trajectory after linear range walk correction by SAR processing, and Fig. 7(b) is the one corrected by the proposed GMTIm method. Applying azimuth FT, the moving target energy can be focused, as shown in Fig. 8. Obviously, Fig. 8(a) is more concentrated than Fig. 8(b), which means that the proposed GMTIm method outperforms the referencing method.

V. CONCLUSION

An improved GMTIm algorithm is proposed in this paper, which is capable of estimating more high-order phase terms in comparison with the existing methods. A constant velocity model is adopted for subaperture processing, since the corresponding CPI is quite small. From the estimation results of subaperture CPI, the instantaneous DF is obtained. By combining the instantaneous DF into a linear system of equation, the polynomial coefficients for whole CPI can be determined by solving a least square problem. The proposed GMTIm algorithm is suitable for high-squint SAR mode and flexible for various polynomial signals. More details about this work will be presented in another paper, including moving target detection, azimuth-dependence of signal phase, and an adaptive-order polynomial estimator.

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