

# Practical Waveforms Coded by Golay Binary Pairs to Achieve Extra-Low Range Sidelobes and Doppler Resilience

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**Abstract**— Phase continuous and differentiable quadriphase waveforms are coded by the Gaussian minimum shift keying (GMSK) scheme using Golay binary complementary sequences. To satisfy spectral requirements, the waveforms are further optimised by a spectrum-optimisation process. The extra-low sidelobe level is achieved by the use of extended mismatched-filter in pulse compression. To improve Doppler tolerance, the Doppler resilient waveforms are derived that provide the first-order suppression to the raised sidelobes introduced by the Doppler frequency shift in the return signal.

**Keywords**—Phase-coded waveform; Doppler resilient; Complementary waveforms; Golay pairs.

## I. INTRODUCTION

As technologies especially digital signal processing (DSP) advance, there is a potential demand for radar replacing (at least for some modes) its traditional recurrent linear or nonlinear frequency modulated (LFM or NLFM) waveforms by non-recurrent non-predictable phase randomly coded waveforms.

To achieve extra-low range sidelobes, Golay pairs have been proposed to code the waveforms for decades [1, 2]. However, these waveforms have undesirable spectra, which prevent their implementation in real radar systems. In addition, these waveforms are also sensitive to Doppler frequency shift introduced by moving targets, causing significant increases in the range sidelobes and destroying the feature of extra-low sidelobes [3, 4].

A Golay complete complementary pair achieves perfect zero sidelobes [5-7]. The original Golay pairs are represented in binary sequences which is further extended to polyphase ones [1, 2]. In relation to phase-coded waveforms, biphasic and polyphase waveforms coded by binary and polyphase complete complementary sequence (CCS) may result in waveforms with discrete phase changes of  $\pi$ ,  $\pi/2$  and other values. Therefore, waveforms directly coded by CCSs are hard to be implemented in radar systems, and other coding schemes need to be sought out. If waveforms are coded by the Gaussian minimum shift keying (GMSK) scheme, the phase is continuous and differentiable, which significantly improves the spectral distribution and make one big step closer to the their implementation in radar systems [8].

Dong [8] proposed an spectrum optimisation model, through which waveforms with constant amplitude, continuous and differentiable phase and confined bandwidth have obtained. By the same processing waveforms coded by the Golay binary pairs are obtained. However, since this kind of optimisation removes / suppresses the high frequency components from the original spectrum that also destroys the feature of the extra-low sidelobes. To overcome, techniques of extended mismatched filters (EMMFs) are used to re-achieve the extra-low sidelobes.

Inspired by Pezeshki *et al.* [3] and Tang *et al.* [4], we derive Doppler resilient waveforms coded by pairs of Golay binary pairs that provide the first-order suppression to the raised sidelobes introduced by the Doppler frequency shift. The Doppler resilient waveforms significantly improve the Doppler tolerance.

It should be pointed out that in implementing complementary waveforms pairs of pulses need to be transmitted and received to fulfil the complementary waveform processing. In practice this may be implemented by either the time-division multiple access (TDMA) or frequency-division multiple access (FDMA) schemes. However, the target signal may undergo temporal decorrelation in the former scheme and spectral decorrelation in the latter, which may degrade the properties of the complementary waveforms and result in sidelobe rising. However, the detailed analysis of this decorrelation issue is out of the scope of this paper. It is assumed in this paper that the pair of complementary waveforms is transmitted and the copy of them reflected by point targets is received, and either the temporal or spectral decorrelation is not considered.

## II. WAVEFORMS CODED BY GOLAY PAIRS

The references [6, 7] provide a complete description of binary Golay pairs (a binary Golay pair is also called a binary complete complementary pair, BCCP) for lengths up to 100, and some formulae for obtaining longer sequences from shorter sequences by recursive constructions. The length of all possible Golay sequences is restricted by [7],

$$N = 2^\alpha 10^\beta 20^\gamma \quad (1)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are non-negative integers. Therefore, the length of BCCPs can be 1, 2, 4, 8, 10, 16, 20, 26, 32, 40, 52, 64 and 80 for  $N < 100$ . Because the aperiodic autocorrelation is symmetrical, if  $\{a, b\}$  is a BCCP, then  $\{\bar{a}, b\}$ ,  $\{a, \bar{b}\}$  and  $\{\bar{a}, \bar{b}\}$  are also BCCPs where  $\bar{a}$  is the reversed sequence of the original sequence  $a$ . The total number of BCCPs for  $N < 100$  is given in Table 1 [6].

Table 1: The total number of BCCPs for lengths shorter than 100

N	1	2	4	8	10	16	20
No of BCCP	4	8	32	192	128	1536	1088
N	26	32	40	52	64	80	
No of BCCP	64	15360	9782	512	184320	102912	

Direct search for long BCCPs could be time consuming. Alternatively, some longer BCCPs can be constructed from using known shorter BCCPs by recursive constructions [6]. The recursive construction rules indicate,

1. A BCCP of length  $2N$  can be constructed from a BCCP of length  $N$ ;
2. Two BCCPs of length  $MN$  can be constructed from a BCCP of length  $N$  and a BCCP of length  $M$ .
3. Two BCCPs of length  $2MN$  can be constructed from a BCCP of length  $N$  and a BCCP of length  $M$ .

In our exercises, we first used exhaustive search to obtain all 128 BCCPs of length 10, and with these seed sequences, various BCCPs of  $N=100$  are generated using the recursive construction Rule 2.

We found that the Quadrphase-coded (QPC) [9] waveforms (with continuous phase) coded by a Golay pair still possess the extra-low sidelobes. However, even though QPC waveforms are phase continuous and have a much better spectral distribution compared to the phase-discontinuous waveforms, its spectrum does not satisfy the radar spectrum engineering criteria (RSEC) specified by the US National Telecommunications and Information Administration (NTIA) [10; Chapter 5].

The spectrum is optimised using a nonlinear optimisation model given in [8]. The model first using the GMSK to code the waveform, and then the waveform is further refined to suppress the undesired spectral sidelobes. The final waveform has constant amplitude, continuous and differentiable phase and confined bandwidth. Figure 1 shows an example of the spectrum of a GMSK-coded QPC waveform by a binary sequence of  $N = 100$  and the spectrum of the associated spectrum optimized waveform. It can be seen that the latter quite satisfies the spectral criteria of NTIA.

Since both the GMSK-coded waveform and the final waveform significantly suppress high frequency components of the original QPC waveform, which in turn destroys the feature of the extra-low sidelobes initially achieved by the QPC waveforms coded by the Golay binary pair. Figure 2 shows the sidelobe levels of the GMSK-coded QPC and the bandwidth-optimised waveforms coded by a Golay binary pair.

It can be seen that the feature of extra-low sidelobes achieved by the complementary waveforms has been greatly degraded. However, the sidelobe level is still much lower than waveforms coded by non-Golay binary sequences.

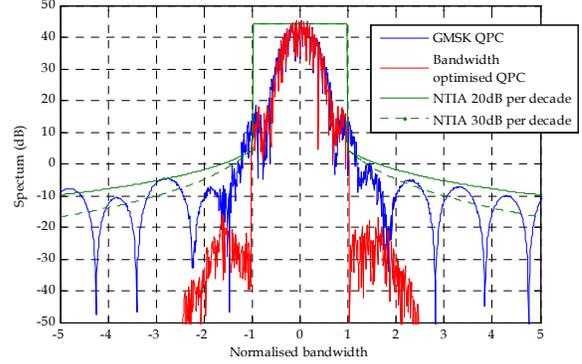


Figure 1: Spectra of the GMSK-coded QPC waveform (the initial waveform, in blue) and the final waveform (in red). The latter well satisfies the spectral criteria specified by NTIA.

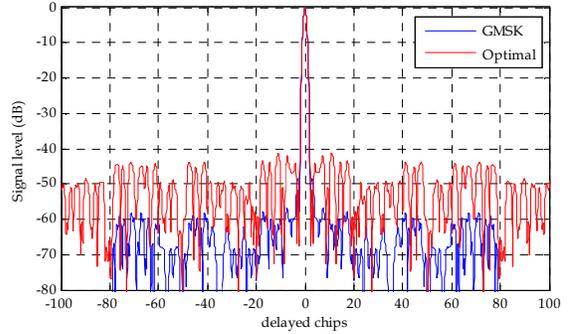


Figure 2: Pairs of the GMSK-coded and the bandwidth-optimised waveforms initially coded by a BCCP lose their complementary property and result in high nonzero sidelobes.

To re-achieve the extra-low sidelobes, the use of EMMF is considered. The forming of the extended mismatched filter in terms of minimising the integrated sidelobe ratio (ISR) is straight forward and the filter coefficients can be found by [11; Chapter 2, 12],

$$\mathbf{s}_{ext} = [\mathbf{A}_{ext}^H \mathbf{W} \mathbf{A}_{ext}]^{-1} \mathbf{A}_{ext}^H \mathbf{W}^H \mathbf{y}'_{ext} \quad (2)$$

where the superscript  $H$  denotes Hermitian transpose,  $\mathbf{s}_{ext} \in \mathbb{C}^{K \times 1}$  is the EMMF and  $K > N$ ; matrix  $\mathbf{A}_{ext} \in \mathbb{C}^{(K+N-1) \times K}$  is loaded by shifted copies of the waveform;  $\mathbf{y}'_{ext} \in \mathbb{C}^{(K+N-1) \times 1}$  is the desired output of the mismatched filter whose central part is the output of the ideal complementary waveforms (e.g. the output of the original QPC waveform with the matched filter) and the remaining parts are assigned to zeros initially; and finally  $\mathbf{W} \in \mathbb{R}^{(K+N-1) \times (K+N-1)}$  is a diagonally loaded weighting matrix, whose pattern is discussed below.

The idea of using the EMMF is to push the unwanted sidelobe energy to the outside of the range of interest to achieve the desired sidelobe suppression for the range of interest. For instance, the maximum range of interest for radar operated in the inverse synthetic radar (ISAR) mode is the

width of pulse to avoid range aliasing. For radar operated in search and detection mode, the range of interest can be longer than the width of pulse, but is still manageable. As an example, Figure 3 shows a design of the weight pattern. As shown, the weight ratio is 100:1 between the range of interest and the outside, resulting in sidelobes for the range of interest to be  $20\log_{10}100 = 40$  dB lower than the outside sidelobes.

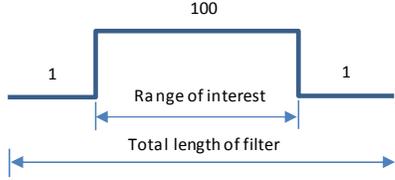


Figure 3: Design of the weight pattern. Since the weight ratio is 100:1 between the range of interest and the outside, the sidelobe level for the range of interest will be 40 dB lower than the outside sidelobes.

The range profile of the bandwidth-optimised waveform shown in Figure 2 is re-processed using the EMMF, and the result is shown in Figure 4. It can be seen that for the desired range of interest, the sidelobe level is suppressed below  $-100$  dB.

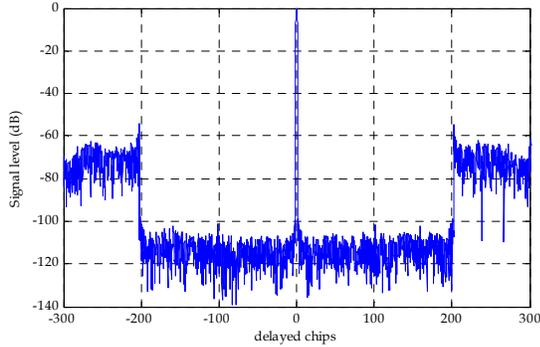


Figure 4: Pulse compression using EMMF for a pair of bandwidth-optimised waveforms ( $N = 100$ ). A PSR  $> 100$  dB is achieved for the desired range of interest.

To demonstrate, a weak target (0 dB signal-to-noise ratio, SNR) located near a strong target (80 dB SNR) is considered, without using the EMMF, the sidelobe of the strong target would mask the weak target, as the waveform can only achieve approximately 40 dB PSR (see Figure 2). However, with the use of the EMMF, sidelobes of the strong target are being pushed low, and the weak target become unmasked as shown in Figure 5. The use of the matched filter (MF) in Figure 5 means that the original extral-low complementary waveforms (which are not implementable due to their unqualified spectra) are used to benchmark the SNR loss the EMMF filter. In fact, we found from rigorous math using the Parseval's theorem (not given here, due to page limit) that the SNR loss of the EMMF was only 0.64 dB.

### III. CONSTRUCTION OF DOPPLER RESILIENT COMPLEMENTARY WAVEFORMS

In the derivation of Doppler resilient complementary waveforms (DRCW) by Pezeshki *et al.* (2008) and Tang *et al.*

(2014), both papers only considered the Doppler shift effect in the slow-time domain, i.e. from pulse to pulse. In other words, they assume that the pulse width is sufficiently short than the pulse repetition interval (PRI) (this is true) and the Doppler shift is small enough so that the Doppler shift effect for the fast-time (within the pulse) is ignored. This is perhaps not a valid assumption. However, the work of Pezeshki *et al.* (2008) and Tang *et al.* (2014) inspires our study.

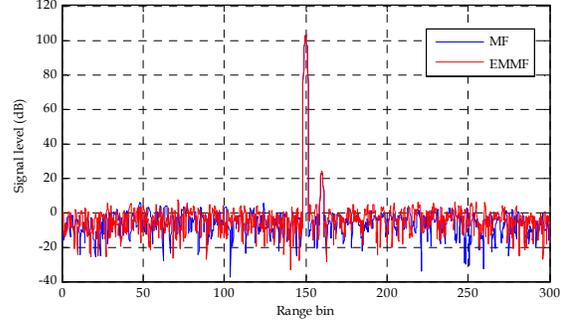


Figure 5: Performance comparison between the matched filter and EMMF: A 80 SNR target located at range bin 150 and a 0 dB SNR target at range bin 160 embedded in bandlimited Gaussian noise. The pulse compression provides a coherent gain of  $10\log_{10}(2 \times 100) = 23.0$  dB to both targets. Sidelobes of both the strong and weak targets have been suppressed to the noise level and the weak target is not affected by the sidelobes of the strong target. There is no noticeable SNR loss for the EMMF relative to the matched filter.

In the following derivation, the Doppler shift effect for both the slow-time and the fast-time is all considered. For a waveform  $s(t)$ , its Fourier transform is,

$$S(f) = \mathbb{F}\{s(t)\} = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft) dt \quad (3)$$

When a Doppler frequency  $-f_d$  is involved, the associated Fourier transform is,

$$S_{f_d}(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f_d t) \exp(-j2\pi ft) dt = S(f + f_d) \quad (4)$$

The spectrum  $S(f + f_d)$  can be expanded into a Taylor series, and its first-order approximation is,

$$S_{f_d}(f) \approx S(f) + S'(f) f_d \quad (5)$$

$$S'(f) = \frac{d}{df} \int_{-\infty}^{\infty} s(t) \exp(-j2\pi ft) dt = -j2\pi \mathbb{F}\{ts(t)\} \quad (6)$$

The pulse compression processing using the matched filter in the frequency domain is given by,

$$\mathbb{F}\{s(t) * s^*(-t)\} = S(f) S^*(f) \quad (7)$$

Note that (3) and (6) can be written as the discrete FFT forms of,

$$S_k = \sum_{n=0}^{N-1} s_n W^{ik} \quad \forall k = 0, \dots, N-1 \quad (8)$$

$$S'_k = -j2\pi \sum_{n=0}^{N-1} n s_n W^{nk} \quad \forall k = 0, \dots, N-1 \quad (9)$$

where  $W = \exp(-j2\pi/N)$ . Since complementary waveforms are coded by Golay pairs (binary or polyphase sequences), we can safely shift our discussions from the pairs of complementary

waveforms  $\{s_a, s_b\}$  and  $\{s_c, s_d\}$  to the pairs of complementary sequences  $\{a, b\}$  and  $\{c, d\}$  in the FFT domain.

Suppose  $\{a, b\}$  is a Golay pair,  $|a_n| = |b_n| = 1$ ,  $n = 0, \dots, N-1$ .  $\{A, B\}$  is the corresponding FFT transformed pair, given by,  $A_k = \sum_{n=0}^{N-1} a_n W^{kn}$  and  $B_k = \sum_{n=0}^{N-1} b_n W^{kn} \quad \forall k = 0, \dots, N-1$ . Since  $\{a, b\}$  is a Golay pair, we have,

$$A_k A_k^* + B_k B_k^* = 2N \quad \forall k = 0, \dots, N-1 \quad (10)$$

Expanding (10) the coefficients for each spectral term  $W^{-ik}$  and  $W^{ik}$ ,  $\forall k = 0, \dots, N-1$ , has to vanish, giving,

$$\sum_{n=0}^{N-1-i} (a_n a_{n+i}^* + b_n b_{n+i}^*) = 0 \quad \forall i = 0, \dots, N-1 \quad (11)$$

$$\sum_{n=0}^{N-1-i} (a_n^* a_{n+i} + b_n^* b_{n+i}) = 0 \quad \forall i = 0, \dots, N-1 \quad (12)$$

The derivatives  $A'_k$  and  $B'_k$ , according to (9), are,

$$A'_k = -j2\pi \sum_{n=0}^{N-1} n a_n W^{kn} \quad \forall k = 0, \dots, N-1 \quad (13)$$

$$B'_k = -j2\pi \sum_{n=0}^{N-1} n b_n W^{kn} \quad \forall k = 0, \dots, N-1 \quad (14)$$

$$\begin{aligned} A'_k A_k^* &= -j2\pi [0a_0 + 1a_1W^k + \dots + (N-1)a_{N-1}W^{(N-1)k}] [a_0^* + a_1^*W^{-k} + \dots + a_{N-1}^*W^{-(N-1)k}] \\ &= -j2\pi \left[ \frac{N(N-1)}{2} + \sum_{i=1}^{N-1} \sum_{n=0}^{N-1-i} n a_n a_{n+i}^* W^{-ik} + \sum_{i=1}^{N-1} \sum_{n=0}^{N-1-i} (n+i) a_n^* a_{n+i} W^{ik} \right] \\ &\quad \forall k = 0, \dots, N-1 \end{aligned} \quad (15)$$

The expression of  $B'_k B_k^*$  is the same as (15) by replacing  $a_n$  with  $b_n$  and  $a_n^*$  with  $b_n^*$ .

Suppose two pairs of pulsed complementary waveforms,  $\{s_a, s_b\}$  and  $\{s_c, s_d\}$  are transmitted at a pulse repetition interval (PRI)  $T_r$ , echoes from a point target with a unit reflectivity and Doppler frequency  $-f_d$  received by radar in the baseband, after the range alignment, will be,  $s_a(t)e^{-j2\pi f_d t}$ ,  $e^{-j2\pi f_d T_r} s_b(t)e^{-j2\pi f_d t}$ ,  $e^{-j2\pi f_d 2T_r} s_c(t)e^{-j2\pi f_d t}$  and  $e^{-j2\pi f_d 3T_r} s_d(t)e^{-j2\pi f_d t}$ , where  $t = 0, \tau_c, \dots, (N-1)\tau_c$ .  $\tau_c = T/N$  is the chip width and  $T$  the pulse width. The four phase terms in the slow-time,  $1$ ,  $e^{-j2\pi f_d T_r}$ ,  $e^{-j2\pi f_d 2T_r}$  and  $e^{-j2\pi f_d 3T_r}$  will be compensated in the coherent Doppler processing by using a temporal steering vector of  $[1 \quad e^{j2\pi f_d T_r} \quad e^{j2\pi f_d 2T_r} \quad e^{j2\pi f_d 3T_r}]$ . Therefore essentially we are dealing with four received waveforms in the fast-time,  $s_a(t)e^{-j2\pi f_d t}$ ,  $s_b(t)e^{-j2\pi f_d t}$ ,  $s_c(t)e^{-j2\pi f_d t}$  and  $s_d(t)e^{-j2\pi f_d t}$  which all have the same Doppler frequency term  $e^{-j2\pi f_d t}$ ,  $t = 0, \tau_c, \dots, (N-1)\tau_c$ . The coherent processing using the matched filter in the frequency domain is to evaluate (under the first-order approximation),

$$S_\Sigma(f) \approx (S_a S_a^* + S_b S_b^* + S_c S_c^* + S_d S_d^*) + (S'_a S_a^* + S'_b S_b^* + S'_c S_c^* + S'_d S_d^*) f_d \quad (16)$$

Suppose we have another Golay pair  $\{c, d\}$  and let  $c = \bar{a}$  and  $d = \bar{b}$ , where the symbols  $\bar{a}$  and  $\bar{b}$  denote the conjugated and

reversed sequences of  $a$  and  $b$ , respectively. For convenience, we call  $\{c, d\}$  the companion Golay pair of  $\{a, b\}$ . The corresponded FFT transformed  $\{c, d\}$  is  $\{C, D\}$ . Similarly we can find  $C_k$ ,  $C'_k$ ,  $D_k$ ,  $D'_k$ ,  $C'_k C_k^*$  and  $D'_k D_k^*$ , giving,

$$\begin{aligned} A'_k A_k^* + B'_k B_k^* + C'_k C_k^* + D'_k D_k^* &= -j2\pi [2N(N-1)] \\ &\quad - j2\pi \sum_{i=1}^{N-1} (N-1-i) W^{-ik} \sum_{n=0}^{N-1-i} (a_n a_{n+i}^* + b_n b_{n+i}^*) \\ &\quad - j2\pi \sum_{i=1}^{N-1} (N-1+i) W^{ik} \sum_{n=0}^{N-1-i} (a_n^* a_{n+i} + b_n^* b_{n+i}) \\ &\quad \forall k = 0, \dots, N-1 \end{aligned} \quad (17)$$

Therefore, according to the properties of a Golay pair given by (11) and (12), coefficients of  $W^{-ik}$  and  $W^{ik}$ ,  $i = 0, \dots, N-1$ , in (17) are all zeros, making all terms of  $W^{-ik}$  and  $W^{ik}$ ,  $i = 0, \dots, N-1$ , to vanish, giving,

$$A'_k A_k^* + B'_k B_k^* + C'_k C_k^* + D'_k D_k^* = -j4N(N-1)\pi \quad (18)$$

$$\begin{aligned} S_\Sigma(f_k) &\approx (A_a A_a^* + B_b B_b^* + C_c C_c^* + D_d D_d^*) + (A'_a A_a^* + B'_b B_b^* + C'_c C_c^* + D'_d D_d^*) f_d \\ &= 4N - j4N(N-1)\pi f_d \\ &\quad \forall k = 0, \dots, N-1 \end{aligned} \quad (19)$$

where  $f_k = \sigma_r / (2\pi) = k / N$ . Not explicitly mentioned, but in the above derivations, the sampling interval was assumed to be  $\Delta t = 1$ , so the discrete frequency is  $f_k = k / N$ . The actual sampling interval is  $\Delta t = \tau_c$ , so the discrete frequency becomes  $f_k = k / (N\tau_c)$  and (19) should be written as,

$$S_\Sigma(f_k) = 4N - j4N(N-1)\tau_c \pi f_d \quad (20)$$

The angle of  $S_\Sigma(f_k)$  is,

$$\angle S_\Sigma(f_k) = \tan^{-1} \left( -\frac{4N(N-1)\tau_c \pi f_d}{4N} \right) = -\frac{2\pi f_d (N-1)\tau_c}{2} \quad (21)$$

In (21)  $\tan^{-1}(x) = x$  is used for  $|x| \ll 1$ . This is exactly the mean phase introduced by the Doppler shift frequency  $-f_d$  for the pulse width of  $N\tau_c$ .

Since  $S_\Sigma(f_k)$  in (20) is independent of  $f_k$  for  $k = 0, \dots, N-1$ . The inversed FFT of (20) results in a Dirac delta function with a peak amplitude of  $4N$  (the imaginary part of (20) is significantly smaller than the real part because of  $\angle S_\Sigma(f_k) \ll 1$ ) and a phase of  $\angle S_\Sigma(f_k)$ , under the first-order approximation.

Obviously the complementary waveforms  $\{s_a, s_b\}$  and  $\{s_c, s_d\}$  coded by the Golay pairs  $\{a, b\}$  and  $\{c, d\}$  shall have the same result.

According to the above derivation, the order of four waveforms  $\{s_a, s_b, s_c, s_d\}$  can be arbitrarily swapped without affecting the result. However, in the reality, there exist temporal, spectral and spatial decorrelations for the target signal as well as clutter. For this reason, it might be better to transmit the complementary waveforms pairs by pairs to reduce the effect of the possible decorrelations as the temporal

and spatial decorrelations for moving targets and clutter alike are highly correlated to time. Also, according to the properties of Golay pairs and the above derivation, if waveforms coded by  $\{a, b, c, d\}$  result in a perfect suppression of sidelobes under the first-order approximation, the change of the sign for any of the sequence still leads to the same result. For instance, the waveforms can also be coded by  $\{-a, b, c, d\}$ ,  $\{a, -b, c, d\}$ ,  $\dots$ ,  $\{-a, -b, -c, d\}$ , etc.

Therefore, to achieve the first-order suppression of sidelobes raised by the introduction of Doppler shifts, the number of pulses have to be  $4M$ , where  $M$  is an integer. To generate non-recurrent waveforms, we can select  $M$  independent Golay pairs, generate the associated  $M$  companion Golay pairs, and code the corresponding  $4M$  phase-coded waveforms. A CPI composed of such  $4M$  pulses provides a perfect suppression of sidelobes under the first-order approximation.

Figure 6 shows the results of the DRCW applied to the bandwidth-optimised waveform with the use of EMMF whose sidelobes are shown in Figure 4. For a zero Doppler target, the PSR reaches over 100 dB. If the Doppler frequency of a moving target is 10 kHz, the PSR reduces to about 30 dB. However, if the DRCW is applied, the PSR improves to approximately 55 dB<sup>1</sup>.

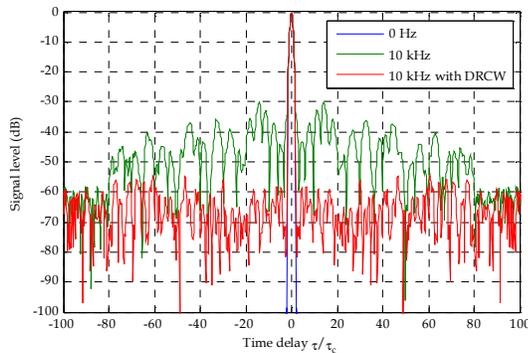


Figure 6: The bandwidth-optimised waveforms with the EMMF in the pulse compressing: (a) PSR is greater than 100 dB for a zero Doppler target (blue curve, also see Figure 4 for details); (b) PSR reduces to about 30 dB for a 10 kHz Doppler frequency target (green curve) and (c) PSR improves to about 55 dB if the DRCM are used (red curve).

#### IV. CONCLUSIONS

A pair of phase continuous QPC waveform coded by a pair of Golay complementary binary sequences achieves zero range sidelobes. However, such waveforms do not satisfy the spectral requirement specified by the US NTIA so that they are unlikely to be implemented in radar systems. We first used the GMSK to code the QPC waveform to have continuous and differentiable phase. The waveform further under goes a spectrum optimisation process to suppress its unwanted

spectral sidebands. The final resultant waveform has constant-amplitude, continuous and differentiable phase and optimised confined bandwidth. However, the removal of high frequency components from the waveforms also destroys their complementary property and results in sidelobe rising.

The technique of EMMF is used to suppress the raised sidelobes. The idea is to push the unwanted sidelobes to a range outside of interest. The resultant sidelobe level can be as low as 100 dB below the mainlobe which is very helpful to mitigate the masking effect generated by strong targets and clutter on weak targets.

The phase randomly coded waveforms have limited Doppler tolerance and the aforementioned waveforms are not immune. Once a moving target introduces a Doppler frequency shift to the received signal, sidelobes increase rapidly by pulse compression. We have derived the Doppler resilient waveforms which normally consist of two pairs of complementary (or quasi-complementary) waveforms, i.e. one pair of complementary (or quasi-complementary) waveforms and a pair of the companion waveforms. The Doppler resilient waveforms provide the first-order suppression to the raised sidelobes introduced by the Doppler frequency shift. The Doppler resilient waveforms significantly improve the Doppler tolerance.

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<sup>1</sup> According to the relationship between Doppler frequency and velocity, a 10 kHz Doppler frequency shift corresponds to a 150 m/s radial velocity for a radar operated at X-band (0.03 m wavelength), and 1250 m/s for a radar operated at L-band (0.25 m wavelength).