MIMO Signal Design for Coexistence with LFM

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***Abstract*—Most of existing waveform optimization works for colocated Multiple-Input Multiple-Output (MIMO) radar focuses on polyphased-coded waveforms, which are discrete and suitable for optimization. In this paper, we optimize MIMO radar waveform design for a radar system that can also simultaneously illuminate continuous Linear Frequency Modulated (LFM) waveforms, such that in the receive end, they have good auto correlation sidelobes, and good cross correlation sidelobes to separate returns caused by different transmit signals. We first samples an LFM signal under a sampling frequency and then formulates a waveform optimization criterion that simultaneously suppress auto correlation sidelobes of MIMO waveforms and cross correlation sidelobes with the sampled discrete LFM signal. Numerical results with a phase coded signal indicate that this method can reach a cross correlation level lower than that between an LFM signal with opposite frequency modulation ratios. Moreover, more phase coded signals are also given to test the method and to investigate the impact of the LFM sampling interval on the final sidelobe level.**

I. Introduction

In the radar field, Linear Frequency Modulated (LFM) signals is widely used for different purposes, such as target detection, Synthetic Aperture Radar (SAR) imaging and jamming toward hostile radar devices. In recent years, colocated Multiple-Input Multiple-Output (MIMO) radar [1], [2] is a hot topic and phased-coded signals, which provides more degrees of freedom for optimization, receives more attention [3], [4]. In some particular situations, one may need to illuminate two kinds of signals simultaneously for certain purpose. In this case, low mutual correlation [5] between two kinds of signals, one discrete and the other continuous, is critical to fulfill two individual tasks. This kind of coexistence in one platform makes particular sense in some situations. But in theory, there are few research existing about how to co-design continuous signals and discrete signals [6], [7].

For the radar function, low range sidelobes (also named as auto correlation sidelobes here) are desirable, in order to keep high-power target returns not submerging lower-power target returns in nearby range cells [8]. Meanwhile, in order to prevent mutual interference between returns caused by two different transmit signals, cross correlation sidelobes [9] should also be suppressed. There are many existing ways to suppress sidelobes of LFM signals and the key is to suppress range sidelobes of phase coded signals. Meanwhile, how to evaluate and suppress cross correlation between continuous LFM signal and discrete phased-coded signals is another challenge. This paper propose a straightforward and simple method to approximate cross correlation between LFM signals and phase coded signals. An LFM signal is first sampled at a sampling interval (or sampling frequency) and then the discrete version can be treated just as other phased coded signals. Consequently, a waveform optimization criterion can be formulated just as how other waveform optimization works do. As an LFM signal has other means to suppress range sidelobes, saying the windowing method, we need not to suppress its auto corrlation sidelobes. That is a difference between classical waveform optimization for colocated MIMO radar [9], [10].

Another difference is that we need to examine whether the discrete LFM signal sufficiently stands for the original continuous LFM signal. In numerical results, we analyze the impact of sampling interval on real cross correlation sidelobes between LFM and phase coded signals. Moreover, in real applications, LFM processing channels often use a window to suppress range sidelobes and this operation would change the expression of cross correlation sidelobes, which is also considered. With range sidelobes involved, it is difficult to find global minimum of the waveform optimization problem and we resort to the Sequential Quadratic Programming (SQP) algorithm [11].

II. Waveform optimization criterion

Consider a radar system composed of several transmit antennas, some of which illuminating an LFM signal and the others illuminating phase coded signals modulated on the same carrier frequency like a colocated MIMO radar. Two kinds of signals are illuminated simultaneously within a pulse but not necessarily of the same duration. In the receive end, there are mainly two channels to deal with received signals with two different range compressing units, one for LFM returns and the other for phase coded signal returns. We expect that not only both channels have low range sidelobes, but also returns due to different probing signals have low outputs (measured by cross correlation sidelobes) in opposite channels for a better separation.

A. Range sideboles of phase coded signals

Assume that there are N_t transmit antennas to transmit phase coded signals, which constitute a transmit waveform matrix as

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_{N_\mathrm{t}}]^\mathrm{T}, \tag{1}$$

where $\mathbf{S} \in \mathbb{C}^{N_\mathrm{t}\times N_\mathrm{s}}$, $\mathbf{s}_i \in \mathbb{C}^{N_\mathrm{s}\times 1}$ denotes the transmit waveform of the ith transmit antenna, $i \in \{1, \cdots, N_\mathrm{t}\}$, N_s denotes the number of codes within each waveform, and

$(\cdot)^{\mathrm{T}}$ denotes the transpose operation. In practice, different signals may be illuminated by different subarrays into different spatial directions, but no further extension about this issue is discussed in this paper. For instance, a subarray may steer its transmit beam toward one target with LFM signals, but another subarray may steer its transmit beampattern toward another target in another direction. This kind of operation can be deemed as a cognitive manner [12], [13], [14] and the valuable time resource can be more efficiently used.

In real applications, most of radar amplification circuits operate in the saturation mode. Therefore, we just consider constant-modulus waveforms here and in this case we can write $\mathbf{S} = \exp(j\mathbf{\Phi})$, where $\mathbf{\Phi} \in [0, 2\pi]$ is the phase matrix of $\mathbf{S}$. Autocorrelation sidelobes of poly-phased coded signal $\mathbf{s}_i$ at range shift k can be expressed as [15].

$$\rho_{i,k} = \mathbf{s}_i{}^{\mathrm{H}} \mathbf{J}_k \mathbf{s}_i / N_{\mathrm{s}}, \tag{2}$$

where $i \in \{1, \cdots, N_{\mathrm{t}}\}, k \in \{1, \cdots, N_{\mathrm{s}} - 1\}$, $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose operator, $\mathbf{J}_k$ here denotes the shift matrix defined by $[\mathbf{J}_k]_{i,j} = \delta_{i+k,j}$, $i, j \in \{1, \cdots, N_{\mathrm{s}}\}$, $k \in \{-N_{\mathrm{s}} + 1, \cdots, N_{\mathrm{s}} - 1\}$, and $\delta_{i,j} = 1$ iff $i = j$ denotes the Kronecker Delta symbol.

In particular, at $k = 0$, $\rho_{i0} = N_{\mathrm{s}}$ holds for constant-modulus signals $\mathbf{s}_i$. Furthermore, $\rho_{i,k}$ meets the following condition,

$$\rho_{i,-k} = \rho_{i,k}^{\mathrm{H}}. \tag{3}$$

Based on this fact, we define an Peak Sidelobe Level (PSL) measure of phase coded signals by

$$\mathrm{PSL}_{\mathrm{a}} = \max_{\substack{k=1,2,\cdots,(N_{\mathrm{s}}-1) \\ i=1,2,\cdots,N_{\mathrm{t}}}} |\rho_{i,k}|, \tag{4}$$

where $|\cdot|$ denotes the absolute value.

B. Sampling and cross correlation sidelobes

The LFM signal with bandwidth B and pulse duration T can be expressed as

$$s_{\mathrm{L}}(t) = \mathrm{rect}(\frac{t}{T}) \exp(j2\pi f_0 t + j\pi\gamma t^2), \tag{5}$$

where $\gamma = B/T$, f_0 denotes the carrier frequency, and

$$\mathrm{rect}(t) = \begin{cases} 1 & t \in [-0.5, 0.5) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

In order to formulate cross correlation sidelobes between LFM signal and phase coded signals, we first need to sample the LFM signal. With a sampling frequency $f_{\mathrm{s}} > B$, we obtain a discrete signal denoted by $\mathbf{s}_{\mathrm{c}} \in \mathbb{C}^{N_{\mathrm{c}} \times 1}$, where $N_{\mathrm{c}} = T f_{\mathrm{s}}$ denotes the length of $\mathbf{s}_{\mathrm{c}}$. Different sampling frequency would result in different lengths of signal $\mathbf{s}_{\mathrm{c}}$. For convenience, we use a sampling frequency f_{s} that would result in $N_{\mathrm{c}} = L N_{\mathrm{s}}$, where L is an integer indicating the sampling manifold. In this case, the code transmission frequency of phase coded signal is $1/L$ times of the presumed sampling frequency over the LFM signal.

The receive end has two kinds of parallel range compression units, one for LFM signal and the other for MIMO signals. we need the range compressors to have low range sidelobes and also returns belong to a channel would output low interference in the other. In order to formulate cross correlation sidelobes in two range compression channels, we denote $\bar{\mathbf{s}}_i = \mathrm{vec}(\mathbf{1}_{L\times 1} \mathbf{s}_i^{\mathrm{T}})$, where $\mathrm{vec}(\cdot)$ denotes the vectorization operation.

In the LFM channel, we often impose a weight denoted by $\mathbf{w} \in \mathbb{R}^{N_{\mathrm{c}} \times 1}$. In this case, sidelobes outputs of phase coded signal returns can be expressed by

$$r_{i,m} = \bar{\mathbf{s}}_i^{\mathrm{H}} \mathbf{J}_m \mathrm{diag}(\mathbf{w}) \mathbf{s}_{\mathrm{c}} / N_{\mathrm{c}}, \tag{7}$$

where $m \in \{-N_{\mathrm{c}} + 1, \cdots, -1, 1, \cdots, N_{\mathrm{c}} - 1\}$, and $\mathrm{diag}(\cdot)$ denotes a diagonal matrix with the entry vector at the diagonal.

In the channel regarding phase coded signals, the sidelobe outputs of LFM returns in this channel are formulated by

$$\bar{r}_{i,m} = \mathbf{s}_{\mathrm{c}}^{\mathrm{H}} \mathbf{J}_m \bar{\mathbf{s}}_i / N_{\mathrm{c}}. \tag{8}$$

Mismatch filter is generally used to further suppress range sidelobes rather than the weighting method [16]. Here we do not use the mismatched filter method.

The Peak Cross-Correlation Level (PCCL) between the encoded signal and the LFM signal can be expressed as

$$\mathrm{PCCL}_{\mathrm{a}} = \max_{\substack{m=-N_{\mathrm{c}}+1,\cdots,-1,1,\cdots,N_{\mathrm{c}}-1 \\ i=1,2,\cdots,N_{\mathrm{t}}}} [|r_{i,m}|, |\bar{r}_{i,m}|]. \tag{9}$$

C. Cross sidelobe level between phase coded signals

Different from $r_{i,m}$ that formulates with $\bar{\mathbf{s}}_i$, cross correlation sidelobes between phase coded signals can be just formulated with $\mathbf{s}_i$, and $\mathbf{s}_j$, by

$$\beta_{i,j,n} = \mathbf{s}_i{}^{\mathrm{H}} \mathbf{J}_n \mathbf{s}_j / N_{\mathrm{s}}, \tag{10}$$

where $i \in \{1, \cdots, N_{\mathrm{t}}\}$, $j \in \{1, \cdots, N_{\mathrm{t}}\}$, $n \in \{-N_{\mathrm{s}} + 1, \cdots, 1, \cdots, N_{\mathrm{s}} - 1\}$, and $i \neq j$. If $i = j$, $\beta_{i,i,n}$ becomes $\rho_{i,k}$.

The Peak Cross-Correlation Level (PCCL) of phase coded signals $\mathbf{s}_i$ and $\mathbf{s}_j$ can be expressed by

$$\mathrm{PCCL}_{\mathrm{c}} = \max_{\substack{n=-N_{\mathrm{s}}+1,\cdots,-1,1,\cdots,(N_{\mathrm{s}}-1) \\ i=1,2,\cdots,N_{\mathrm{t}} \\ j=1,2,\cdots,N_{\mathrm{t}} \\ i\neq j}} |\beta_{i,j,n}|. \tag{11}$$

D. Waveform optimization criterion

According to formulae (4), (9) and (11), we obtain a waveform optimization criterion as

$$\min_{\mathbf{\Phi}} \left[\mathrm{PSL}_{\mathrm{a}}, \alpha \mathrm{PCCL}_{\mathrm{a}}, \beta \mathrm{PCCL}_{\mathrm{c}}\right], \tag{12}$$

where $\alpha, \beta \in (0, +\infty)$ are weights to balance three pursuits. Once $\mathbf{\Phi}$ is obtained, one can obtain $\mathbf{S} = \exp(j\mathbf{\Phi})$. One may impose a constraint $\mathbf{\Phi} \in [0, 2\pi]$, but it is unnecessary.

As range sidelobes are involved, the waveform optimization criterion has many local minima and thus the global minimum is hard to reach. For such an optimization problem, there are many optimization tools and we resort to the Sequential Quadratic Programming (SQP) based algorithm, which can reach a satisfactory result in a short time.

III. Numerical results

A. Simulation configurations

Assume that the bandwidth of an LFM signal is 10MHz and the pulse duration is 12.8μs. We first run the optimization problem with a sampling rate 20MHz. In this case, the code length of LFM after sampling is N_c is 256 and we also assume code length of phase coded signals is $N_\mathrm{s} = 256$. Here auto correlation sidelobe level is measured by APSL in dB by

$$\mathrm{APSL} = 20\log_{10}(\mathrm{PSL_a}). \tag{13}$$

Cross correlation sidelobe level between LFM and phase coded waveforms is measured in a dB form as

$$\mathrm{APCCL_a} = 20\log_{10}(\mathrm{PCCL_a}). \tag{14}$$

Cross correlation sidelobe level of phase coded waveforms are measured in a dB form by

$$\mathrm{APCCL_c} = 20\log_{10}(\mathrm{PCCL_c}). \tag{15}$$

The following optimization results are the best of 5 optimization runs with different initial values. We also set $\alpha = 1$ and $\beta = 1$. We first run the optimization with $N_\mathrm{c} = 1$ to indicate that phase coded signals can achieve a better orthogonality than an LFM with opposite frequency modulation ratio. Next we run the optimization process with more phased-coded signals.

B. Correlation between the LFM signal and a phase coded signal

LFM signals with positive and negative frequency modulation ratios of the same magnitude are often deemed to be orthogonal. But in fact, the orthogonality even at different mutual delays is not strict, since their cross correlation sidelobes are not zeros but just at a low level. We first consider a case with only one phase coded signal, i.e., $N_\mathrm{t} = 1$, and then run the optimization process. Cross correlation sidelobes between two LFM signals with the same frequency modulation ratio but in opposite values are shown in Fig. 1(a), indicating that the peak sidelobe level is about -21.5dB. That is a low value, but still higher than our optimization results, shown in Fig. 1(b), where the peak cross correlation sidelobe level decreases by 4.8dB to -26.3dB. As auto correlation and cross correlation sidelobes are imposed with the same weight, range sidelobe level of the phase coded signals is also -26.3dB.

Our method optimizes the LFM signal to fit into existing optimization tools and it is reasonable to infer that the sampling frequency would have a great impact on real cross correlation sidelobe level. With a much smaller sampling interval, we can investigate real cross correlation sidelobes between the LFM and the phase coded signal. After several numerical simulations, we find that a 80MHz sampling frequency is sufficient to generate a stable cross sidelobe level, which is shown in Fig. 1(c). It can be found that the $\mathrm{APCCL_a}$ increases by 1.8dB to -24.5dB, but still lower than cross correlation sidelobe between the two LFM waveforms of opposite frequency modulation ratios.

This result indicates that phase coded signals can reach a lower correlation with continuous LFM signals. Moreover, orthogonality through frequency modulation ratio is hard to achieve when more than two nearly orthogonal waveforms are required. In this case, phase coded signal become a better choice.

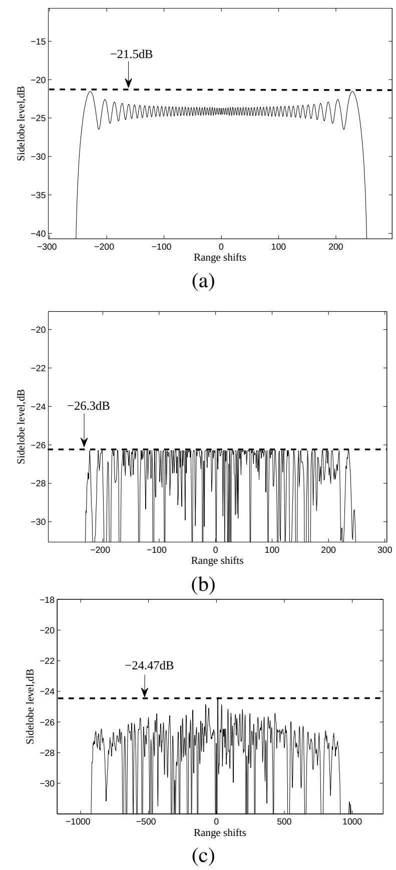
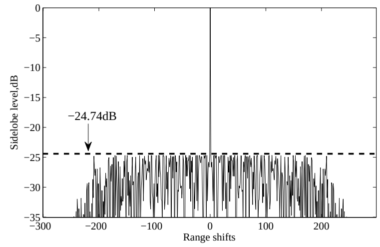


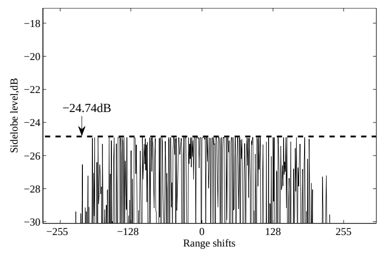
Fig. 1. (a) Cross correlation sidelobes between LFM signals with opposite frequency modulation ratios; (b)Cross correlation sidelobes between a phase coded signal and LFM signal after optimization; (c) Cross correlation sidelobes between phase coded signals and the LFM sampled at frequency 80MHz.

C. LFM signals and more phase coded signals

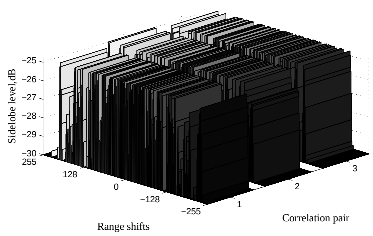
Now we consider another case with more (3 exactly) phase coded signals. Keeping other simulation parameters the same, we run the optimization process and then obtain Fig. 2, where Fig. 2(a) shows range sidelobes of a phase coded signal, Fig. 2(b) shows cross correlation sidelobes of phase coded signals and Fig. 2(c) shows cross correlation sidelobes between three phase coded signals and the LFM signal after optimization. It can be seen that with more phase coded signals, the APSL measures increases by 1.6dB to -24.7dB. In contrast to -26.3dB for the single phase coded signal case, it is still lower than cross correlation between the two LFM signals of opposite frequency modulation ratios.



(a)



(b)



(c)

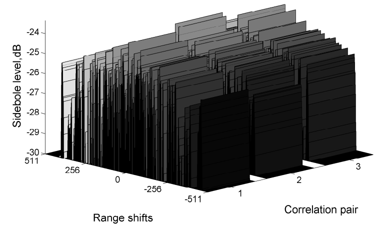
Fig. 2. (a) Autocorrelation sideboles of phase coded signals; (b)Cross correlation sidelobes between phase coded signals; (c)Cross correlation sidelobes of phase coded signals and LFM signal.

D. The effect of signal sampling interval

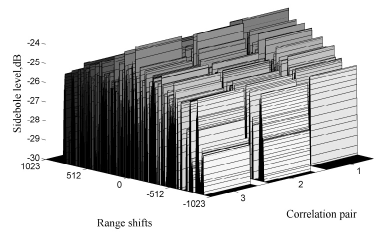
Fig. 2 shows sidelobes at optimization ticks, which may disagree with real cross correlation. To study real cross correlation sidelobes, we present Fig. 3, where Fig. 3(a) samples the LFM at 40MHz and Fig. 3(b) samples it at 80MHz. It can be seen again that real $\mathrm{APCCL_a}$ would increase, by 1.38dB to -23.36dB in Fig. 3(a) and by 1.65dB to -23.09dB in Fig. 3(b). The magnitude is higher than the simulation result in a phase coded signal, but still lower than cross correlation between two LFM waveforms. That can be deemed as an advantage of this method.

E. Optimization results with a window on LFM

In practice, LFM channel often uses a window to suppress range sidelobes. Running the optimization processing with the same settings but imposing a Hamming window, we obtain cross correlation sidelobes between LFM and three phase coded signals, as shown in Fig. 4, where the $\mathrm{APCCL_a}$ decreases 1.84dB from Fig. 2(c) to -26.58dB as a result of optimization. Consequently, it is helpful to consider real signal processing operations for a lower sidelobe level of both auto and cross correlation sidelobes.



(a)



(b)

Fig. 3. (a) Cross correlation range sidelobes of phase coded signals and LFM signals under double samping. (b) Cross correlation sidelobes of phase coded signals and LFM signals under 4 times samping.

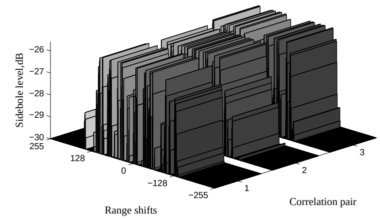
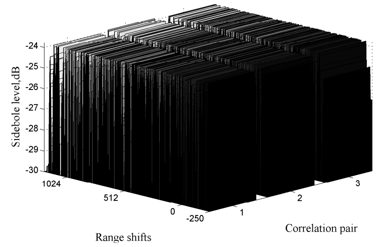


Fig. 4. Cross correlation sidelobes between phase coded signals and LFM with a window.

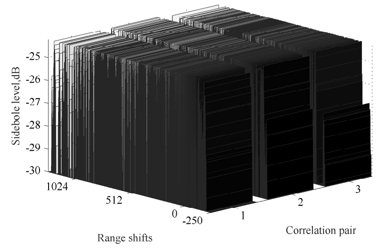
F. Optimization results with different pulse durations or different bandwidth

In practice, it may be desirable that LFM signal and phase coded signal have different pulse durations. The Fig. 5(a) considers a case where the LFM signal has a 64μs pulse duration. The bandwidth is still 10MHz and the sampling frequency is 20MHz, yielding 1280 sampling points. The Fig. 5(b) considers the other case where the LFM signal has a 50MHz bandwidth and the pulse duration is still 12.8μs, also yielding 1280 sampling points. In both cases, three phase coded signals still have the same pulse duration 12.8μs and the same number of codes (i.e., $N_\mathrm{c} = 256$). The LFM signals and three phase coded signals are aligned to the same center. In both cases, LFM signal has 5 times of codes in phase coded signals. Fig. 5(a) and Fig. 5(b) show cross correlation sidelobes between LFM and three phase coded signals in two cases. It can be seen from Fig. 5(a) that the $\mathrm{APCCL_a}$ increases by 0.83dB in Fig. 2(c) to current -23.81dB. And in Fig. 5(b), the $\mathrm{APCCL_a}$ increases by 0.55dB in Fig. 2(c) to current -24.15dB.

In Figs. 4-5, we show more results on cross correlation sidelobes between LFM and phase coded signals. Range sidelobes of phase coded signals has the same sidelobe level due to the equal weights on auto and cross correlation sidelobes. Moreover, the sampling rate on LFM would not effect auto correlation sidelobes of phase coded signals.



(a)



(b)

Fig. 5. (a) Cross correlation sidelobes of phase coded signals and LFM signal with longer pulse durations. (b) Cross correlation sidelobes of phase coded signals and LFM signal with longer bandwidth.

In all, we summarize the findings as follows.

- First, phased-coded signals can achieve a lower cross correlation level with LFM signals than an LFM with opposite frequency modulation ratios;
- Second, more phase coded signals will deteriorate the final sidelobe level;
- Third, a window on LFM would be helpful to a lower sidelobe level;
- Forth, longer time-bandwidth product of LFM signal would lead to worse sidelobe level.

IV. Conclusion

In this paper we consider how to design phase coded waveforms for an array radar system that also illuminate LFM signals simultaneously. In theory, waveform optimization involving both continuous and discrete signals is scare in existing research. The emphasis is placed to how to reduce cross correlation between LFM and phase coded signal, in order to make a receive end to separate returns caused by two different signals. Meanwhile, phase coded signals are designed for low range sidelobes to facilitate radar target detection. The basic concept is to sample the LFM signal first and then express the waveform optimization problem with discrete codes. The impact of sampling frequency on LFM signal is studied via numerical results, indicating that this method works well, i.e., one can control the final sidelobe level through controlling the sampling frequency. Moreover, this method is shown capable of achieving a cross correlation sidelobe level lower than two LFM signals with opposite frequency modulation ratios.

Multi-function at a single platform is interested in many situations. Considering wide applications of LFM signals, we carry out this study at the background of colocated MIMO radar and cognitive radar. This is just our initial works about this issue. In future, the Doppler issue, which in real applications is inevitable, will be taken into account. In any way, this works explores the possibility of coexistence between LFM signal and phase coded signals and more efforts would be made on this issue.

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