Adaptive Detection with Censored Data in Multistatic Radar

Ding Cao, Shenghua Zhou, Hongwei Liu, Jibin Zheng
National Laboratory of Radar Signal Processing
Xidian University
Xi'an, China
dingcao@xidian.edu.cn, shzhou@mail.xidian.edu.cn

Xiaoyang Xie
China Academy of Launch Vehicle Technology
Beijing, China

***Abstract*—In multistatic radar, there always exist communication constraints between local radar sites and a fusion center (FC). In this paper, a censored data-based decentralized detection approach is proposed by deriving generalized likelihood ratio test (GLRT) to decrease the need for communication bandwidth. A coherent multi-channel array is utilized at each receiver where only local test statistics (LTSs) exceeding a local threshold are transmitted. The FC makes a global decision through noncoherent integration of local decisions, where the closed-expression for the probability of false alarm (PFA) is derived given communication constraints. Theoretical results are confirmed with Monte-Carlo simulations; Meanwhile, it is demonstrated that the proposed method would maintain better performance than "OR" rule while saving more bandwidth.**

Keywords—multistatic radar; censored data; generalized likelihood ratio;

I. INTRODUCTION

Multistatic radar, also termed as multisite radar system or distributed radar, has been a hot topic since radar was invented [1]. Inspired by the concept of multiple-input multiple-output (MIMO) in communication, the distributed MIMO radar has also attracted intensive attentions in recent years. In [2], it can be treated as a multistatic system with widely separated antennas, which again promotes the development of data fusion. Due to the diversity gain, it shows advantages over traditional monostatic radar in many aspects, e.g. target detection and parameter estimation. As a result, one of the vital problems in multistatic radar is to efficiently fuse local observations for global decision making.

Distributed detection can be categorized into two cases [3]: one is to utilize raw observations for data fusion; the other is to make data compression before combining local decisions. Due to no information loss, the former method, also called the centralized fusion, has a potential to achieve the optimum detection performance in some criterion, e.g. Neyman-Pearson criterion. The latter one is termed as the decentralized detection, which alleviates the need for communication bandwidth. As the development of communication, it is convenient to relax communication constraints using wire transport technologies, e.g. fiber optic technique. However, it is still important to complete data fusion with low communication rate for improving the flexibility in deployment of radar sites. Decentralized detection with quantized decisions is one of the most canonical technologies. To some extreme cases, local radar site would transmit one-bit hard-decision to the FC for further fusion [3]. However, such method would bear enormous information loss since the raw observation is quantized into only two levels. In order to improve the detection performance, multi-level quantization has been considered in the case where transmission channels would have greater transmission capacity [4-5].

Many data compression technologies are at the background of communication. In the area of multistatic radar, it is always neglected that targets only occupy a fraction of range cells in surveillance space, which is true for some kind of radar, e.g. ground-to-air radar. Most communication bandwidth is wasted on transmitting noise. Hence, frequent communication between local radar sites and a FC is not necessary. Inspired by the difference of statistical characteristics between null (H_0) and alternative (H_1) hypotheses, a local threshold can be set at the local radar sites to prevent transmitting local test statistics (LTSs) of very small values, where the communication constraints can be satisfied through adjusting local thresholds.

In this paper, we consider a mutistatic radar system where each receiver possesses a coherent multi-channel array. Each local radar site calculates a LTS using the generalized likelihood ratio test (GLRT), and transmits only when the LTS exceeds a given local threshold. Under independence assumption, the FC receives censored data, and then makes a global decision through noncoherent integration of local decisions. The closed-form expression is derived for global probability of false alarm (PFA).

II. SIGNAL MODEL

In this section, a signal model of multistatic radar is presented with widely separated radar sites. Suppose that a multistatic radar consists of N_t transmitters and N_r receivers. Hence, $N = N_t N_r$ transmit-receive paths, called the spatial diversity channels (SDCs), are obtained at the receive end by designing orthogonal waveforms [6]. Such orthogonality may be imposed in frequency domain or time domain. In the context

This work is partially supported by the National Natural Science Foundation of China (61401329，61271291，and 61501351) and the National Science Fund for Distinguished Young Scholars (61525105).

of this paper, it is assumed that the ideal orthogonality among different transmit signals is maintained even for different mutual delays such that

$$\frac{1}{E}\int_t s_v(t)\, s_w^{\dagger}(t+\tau)\, dt = \begin{cases} 1 & v = w, \tau = 0, \\ 0 & otherwise, \end{cases} \tag{1}$$

where $s_m(t)$ is the mth transmitter's transmit signals for $m = 1,2,\cdots,N_t$, E is energy of transmit signals, and superscript † stands for conjugate transpose. Further suppose that the independent observations can be realized by the geographical separation of the receivers. As a result, N independent observations are obtained at the FC.

In real application, it is common that each receiver possesses a multi-channel system with $N_{a,i}$ receive antennas where $i = (n-1)N_t + m$ is an index of SDCs for $n = 1,2,\cdots,N_r$. Through slow-time sampling, the radar waveform is a simple burst of identical pulses, say $N_{p,i}$ in number. This results in a total of $Q_i = N_{a,i}N_{p,i}$ complex pulses for an individual range gate. The test data (primary data) under H_0 or H_1 can be expressed as

$$\begin{cases} H_0 : \mathbf{z}_i = \mathbf{w}_i \\ H_1 : \mathbf{z}_i = b_i\mathbf{s}_i + \mathbf{w}_i \end{cases} \tag{2}$$

where

- The noise vector $\mathbf{w}_i$ corrupting the ith SDC's data is assumed to be circularly symmetric complex Gaussian distributed, i.e., $\mathbf{w}_i \sim \mathcal{CN}(0,\mathbf{M}_i)$, where $\mathcal{CN}(\bullet)$ stands for circularly symmetric, complex Gaussian distribution, and $\mathbf{M}_i$ is a positive definite covariance matrix of dimension Q_i .
- $\mathbf{s}_i$ is a space-time steering vector relative to the specific propagation geometry of the ith SDC.
- b_i is a deterministic but unknown complex amplitude of target echo.

The data from reference cells (secondary data), which are free of target component, are usually assumed to be independent and identical distributed (i.i.d.). Suppose that the number of reference cells corresponding to the ith SDC is K_i. The data of the jth reference cell, denoted by $\mathbf{z}_{i,l}$ for $l = 1,2,\cdots,K_i$, also follow the circularly symmetric complex Gaussian distribution, i.e., $\mathbf{z}_{i,l} \sim \mathcal{CN}(0,\mathbf{M}_i)$. All the measurements are hence governed by

$$\begin{cases} H_0 : \begin{cases} \mathbf{z}_i \sim \mathcal{CN}(0,\mathbf{M}_i) \\ \mathbf{z}_{i,l} \sim \mathcal{CN}(0,\mathbf{M}_i) \end{cases} \\ H_1 : \begin{cases} \mathbf{z}_i \sim \mathcal{CN}(b_i\mathbf{s}_i,\mathbf{M}_i) \\ \mathbf{z}_{i,l} \sim \mathcal{CN}(0,\mathbf{M}_i) \end{cases} \end{cases} \tag{3}$$

The probability density function (PDF) under H_k for $k = 0,1$ can be denoted by

$$\begin{cases} f(\mathbf{z}_i | H_k) = \dfrac{1}{\pi^{Q_i}\|\mathbf{M}_i\|}\exp\left[(\mathbf{z}_i - kb_i\mathbf{s}_i)^{\dagger}\mathbf{M}_i^{-1}(\mathbf{z}_i - kb_i\mathbf{s}_i)\right] \\ f(\mathbf{z}_{i,l} | H_k) = \dfrac{1}{\pi^{Q_i}\|\mathbf{M}_i\|}\exp\left(\mathbf{z}_{i,l}{}^{\dagger}\mathbf{M}_i^{-1}\mathbf{z}_{i,l}\right) \end{cases}, \tag{4}$$

with $\|\bullet\|$ denoting the determinant of a matrix.

III. Adaptive Detection with Censored Data

Although the Neyman-Pearson criterion achieves the optimum performance, it is difficult in real application to obtain some parameter under H_1, such as signal-to-noise (SNR). A common and suboptimum method is to utilize GRLT at the FC where the fusion detector is derived by substituting unknown parameters with their maximum likelihood estimates [7]. Let $\mathbf{Z}_i$ be a matrix that comprises all measurements received by the ith SDC such that $\mathbf{Z}_i = \left[\mathbf{z}_i\ \mathbf{z}_{i,1}\ \cdots\ \mathbf{z}_{i,K_i}\right]$. In centralized system, the $\mathbf{Z}_i$s are transmitted directly to the FC for global decision. Under independence assumption, the GLRT with raw observations (GLRT-R) for a multistatic system can be denoted by [8-10]

$$\begin{aligned} T_R &= \log\frac{\max_{\{b_i,\mathbf{M}_i | i=1,2,\cdots,N\}} f(\mathbf{Z}|H_1)}{\max_{\{\mathbf{M}_i | i=1,2,\cdots,N\}} f(\mathbf{Z}|H_0)} \\ &= \sum_{i=1}^{N}\log\frac{\max_{\{b_i,\mathbf{M}_i\}} f(\mathbf{Z}_i|H_1)}{\max_{\{\mathbf{M}_i\}} f(\mathbf{Z}_i|H_0)} = \sum_{i=1}^{N} x_i \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \end{aligned} \tag{5}$$

where η is a global threshold corresponding to a given PFA, and x_i is the LTS from the ith SDC. As shown in [7], x_i can be written as

$$x_i = (K_i + 1)\log\left(\frac{1 + \mathbf{z}_i^{\dagger}\hat{\mathbf{M}}_i^{-1}\mathbf{z}_i}{1 + \mathbf{z}_i^{\dagger}\hat{\mathbf{M}}_i^{-1}\mathbf{z}_i - \dfrac{\left|\mathbf{s}_i^{\dagger}\hat{\mathbf{M}}_i^{-1}\mathbf{z}_i\right|}{\mathbf{s}_i^{\dagger}\hat{\mathbf{M}}_i^{-1}\mathbf{s}_i}}\right) \tag{6}$$

where $\hat{\mathbf{M}}_i = \sum_{l=1}^{K_i}\mathbf{z}_{i,l}\mathbf{z}_{i,l}^{\dagger}$ involves only the secondary data which follows the well-known Wishart distribution. A condition that we impose here is $K_i \geq Q_i$ such that the matrix $\hat{\mathbf{M}}_i$ is nonsingular with probability one. From (5), both the denominator and numerator can be maximized individually for each SDC.

In some scenario, the communication bandwidth between local radar sites and the FC may not afford transmitting raw observations or LTSs. It is an efficient method to utilize censored data to alleviate the demand for communication bandwidth where only the most "informative" data are transmitted [11-12]. Assume that targets dominate only a fraction of range cells in surveillance space. Most of the communication bandwidth is wasted on transmitting just noise. Censoring scheme may decrease the communication load while guaranteeing a certain level of performance. By virtue of the difference between the statistical characteristics of the observations under H_0 and H_1, the core of such method is to utilize a local threshold to discard some "uninformative" LTSs. Therefore, the local communication rate can be denoted by [10]

$$\alpha_i = \Pr\left(x_i > \eta_i \middle| H_0\right) \tag{7}$$

where $\alpha_i \in [0,1]$ is a constraint on communications per resolution cell, and η_i is the local threshold for the ith SDC. For example, we consider a scenario where the sampling frequency is f_s=2 MHz, the maximum detection range is $R_{\max} = 450$ km, and the pulse repetition interval is $T_r = 3$ ms. Suppose that a faithful rendition of local observation needs $q = 12$ bits. For centralized detection, the communication rate requires at least

$$Q = \frac{2R_{\max} f_s q}{cT_r} = 2.86 \text{ MB/s}. \tag{8}$$

where c is a constant denoting the light velocity. Given $\alpha_{0,i} = 0.01$, the communication constraint decreases to $\alpha_{0,i}Q = 29.29$ kB/s, where the local threshold η_i controls the communication rate such that only $\alpha_i \times 100\%$ of all data are transmitted in a scanning cycle. Hence, the local decision rule can be written as

$$u_i = \begin{cases} x_i, & x_i \ge \eta_i \\ q_i, & x_i < \eta_i \end{cases} \tag{9}$$

where q_i is a discrete decision for $x_i \in [0, \eta_i)$. It is easy to verify that we have $q_i = 0$ based on GLRT [5]. From (9), the LTS u_i has both continuous and point-mass characteristics. Considering randomization decision [12], the global decision rule can be written as

$$z = \sum_{i=1}^{N} u_i \begin{cases} > \eta, & \text{decide } H_1 \\ < \eta, & \text{decide } H_0 \\ = \eta, & \text{decide } H_1 \text{ with probability } \gamma \end{cases} \tag{10}$$

where η is a global decision threshold to control the PFA, $\gamma \in [0,1]$ is a randomization parameter to satisfy given PFA if possible, z is the global test statistic. The topology of GRLT with censored data (GLRT-C) is shown in Fig.1.

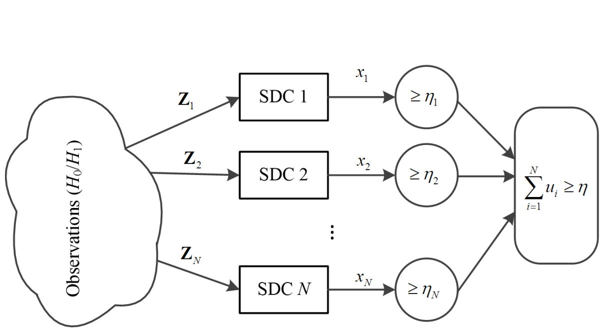

Fig. 1. Topology of GLRT-C

IV. False Alarm Rate

For a multistatic system with N SDCs, the global PDFs under H_0 can be derived by N-fold convolution, which can be written in the form of

$$f_0(z) = g_0(z)\varepsilon(z - \eta_{\min}) + \prod_{i=1}^{N}(1-\alpha_i)\delta(z) \tag{11}$$

where $g_0(z)$ is a deterministic but unknown function, $f_0(z)$ is the PDF of global test statistic, and $\eta_{\min}$ can be written as

$$\eta_{\min} = \min_{i=1,2,\cdots,N} \eta_i \tag{12}$$

By virtue of randomization decision rule, the PFA P_0 can be derived as

$$P_0 = \Pr\left(z \ge \eta \middle| H_0\right) = \int_{\eta}^{+\infty} f_0(z)\,dz + \gamma \Pr\left(z = \eta \middle| H_0\right). \tag{13}$$

Due to the point-mass characteristics, multistatic radar system may not satisfy any given PFA just through adjusting global threshold. The relationship between PFA and the global threshold can be categorized into two cases.

Case 1: $\eta > 0$. From (11), the global test statistic has continuous characteristics for $z \in [\eta, +\infty)$, which leads to $\Pr\left(z = \eta \middle| H_0\right) = 0$. Therefore, (13) can be rewritten as

$$P_0 = \Pr\left(z \ge \eta \middle| H_0\right) = \int_{\eta}^{+\infty} g_0(z)\,dz, \tag{14}$$

Despite the difficulty in calculating $g_0(z)$ directly, the conditional probability is much easier to derive given all SDCs' transmission states. Define S_j to represent one of all SDCs' transmission states for $j = \sum_{i=1}^{N} b_i 2^{i-1}$, where $b_i = 1$ if the ith SDC transmits, and $b_i = 0$ otherwise. The PFA can be rewritten as a sum version of conditional probabilities, i.e.,

$$P_0 = \sum_{j=1}^{2^N-1} P_0(S_j) \int_{\eta}^{+\infty} g_0(z|S_j)\,dz = \sum_{j=1}^{2^N-1} P_0(S_j) P_0^j, \tag{15}$$

where $P_0(S_j)$ denotes the occurrence probability of event S_j, $g_0(z|S_j)$ is the conditional PDF given S_j, and P_0^j is the conditional PFA. As for the local test statistic x_i in (6), Kelly [7] has verified that it follows the exponential distribution with mean $(K_i+1)/(K_i+1-Q_i)$ under H_0, i.e.,

$$f\left(x_i \middle| H_0\right) = \frac{K_i+1-Q_i}{K_i+1} \exp\left[-\frac{(K_i+1-Q_i)x_i}{K_i+1}\right]. \tag{16}$$

Hence, α_i can be written as

$$\alpha_i = \exp\left[-\frac{(K_i+1-Q_i)x_i}{K_i+1}\right] \tag{17}$$

Moreover, $P_0(S_j)$ and P_0^j can be written as

$$P_0(S_j)=\prod_{i:x_i>\eta_i}\alpha_i\prod_{i:x_i<\eta_i}(1-\alpha_i) \tag{18}$$

and

$$P_0^j=\Pr\left(\sum_{i:x_i>\eta_i}x_i>\eta\middle|S_j,H_0\right), \tag{19}$$

respectively. The conditional PDF of x_i in (19) can be denoted by

$$\begin{aligned}f_i(x_i|S_j)&=\frac{f_i(x_i)}{\alpha_i}\varepsilon(x_i-\eta_i)\\&=\frac{K_i+1-Q_i}{K_i+1}\exp\left[-\frac{(K_i+1-Q_i)(x_i-\eta_i)}{K_i+1}\right]\varepsilon(x_i-\eta_i).\end{aligned} \tag{20}$$

which still follows the exponential distribution. Therefore, (19) can be simplified to

$$\begin{aligned}P_0^j&=\Pr\left(\sum_{i:x_i>\eta_i}(x_i-\eta_i)>\eta-\sum_{i:x_i>\eta_i}\eta_i\middle|S_j,H_0\right)\\&=1-F_{sum}\left(\eta-\sum_{i:x_i>\eta_i}\eta_i\right),\end{aligned} \tag{21}$$

where F_{sum} denotes the cumulative distribution function (CDF) of a sum of exponential variates governed by (16). Especially, $F_{sum}(\cdot)$ is the CDF of the gamma distribution when the number of secondary data and the signal vector length are the same among all SDCs.

Case 2: $\eta=0$. In this case, we have $\Pr(z=\eta|H_0)>0$. Therefore, the PFA denoted by (13) can be simplified to

$$\begin{aligned}P_0&=\int_0^{+\infty}f_0(z)dz+\gamma\Pr(z=0|H_0)\\&=1-(1-\gamma)P_0(S_0),\end{aligned} \tag{22}$$

where $P_0(S_0)$ can be denoted by

$$P_0(S_0)=\prod_{i=1}^{N}(1-\alpha_i). \tag{23}$$

As for a given PFA larger than $P_0(S_0)$, decreasing the global threshold cannot satisfy the desired PFA anymore. However, censoring scheme may achieve any PFA through setting an appropriate randomization parameter γ.

It should be noticed that the detection performance can be analyzed in the same method following (13)-(21). However, it is not easy to achieve the closed-form expression of F_{sum} under H_1 currently, which needs further investigation.

V. SIMULATION RESULTS

In this section, numerical results are conducted to verify the effectiveness of the proposed method. For simplicity, we consider a multistatic system with two transmitters and two receivers. As a result, there are four SDCs for data fusion. Suppose that all SDCs have the same vector length where $Q_1=Q_2=Q_3=Q_4=10$, and the numbers of secondary data are $K_1=K_2=K_3=K_4=20$.

In order to examine the PFA of proposed detector, we consider three cases with different local communication rates.

Case 1: $\alpha_1=1,\ \alpha_2=1,\ \alpha_3=0.05,\ \alpha_4=0.05$, i.e., the first receiver not only receives signals, but also acts as a FC.

Case 2: $\alpha_1=0.01,\ \alpha_2=0.01,\ \alpha_3=0.01,\ \alpha_4=0.01$, i.e., all communication constraints are identical.

Case 3: $\alpha_1=0.002,\ \alpha_2=0.004,\ \alpha_3=0.006,\ \alpha_4=0.008$, i.e., communication constraints are distinct among all SDCs.

The PFA as a function of global threshold for three cases is reported in Fig. 2, where the dashed, solid and dashed-dotted lines indicate the results obtained from (13)-(21) for Case 1, 2, and 3, respectively. As for Case 1, the global test statistic has only continuous characteristics. It can achieve any PFA through adjusting global threshold. For Case 2 and 3, the global test statistics have both continuous and point-mass characteristics. Although randomization decision may achieve better detection performance, we preferred to set $\gamma=0$ in real application since randomization decision occurs only when the FC receives no data. In this scenario, H_1 is chose randomly with probability γ, which makes a decision with no further information. Hence, horizontal lines are plotted when the PFA is larger than $1-P_0(S_0)$. The symbols represent the corresponding results obtained from the Monte-Carlo simulations. The number of independent trials is 10^5. It is shown that the theoretical results match the Monte-Carlo simulations pretty well.

The probabilities of detection (PDs) of three cases are shown in Fig. 3. No antenna alignment errors are considered here. Therefore, target steering vectors were assumed to be accurate. Without loss of generality, assume that the output SNRs a_is are the same among all SDCs, which can be denoted by

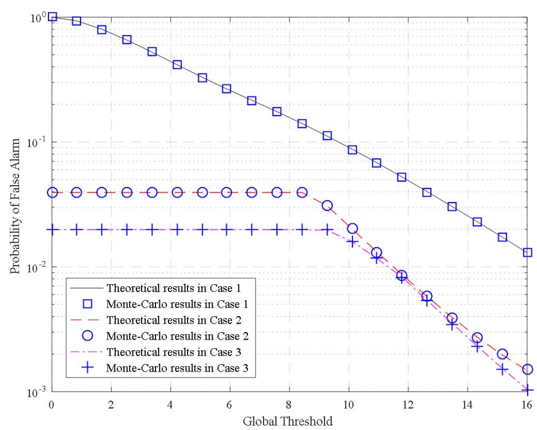

Fig. 2. PFA versus global thresholds in three cases.

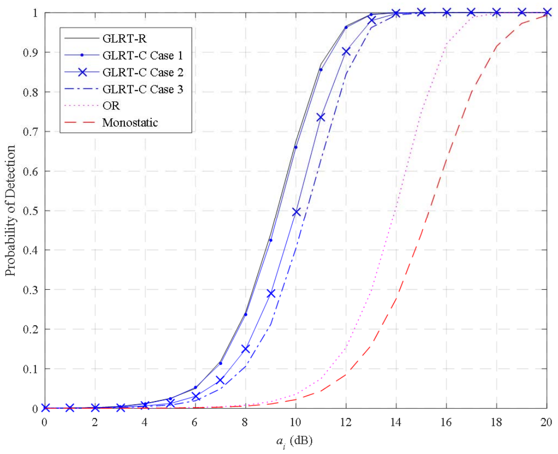

Fig. 3. Detection performance of GLRT-C for three cases.

$$a_i = |b_i|^2 \mathbf{s}_i^\dagger \mathbf{M}_i^{-1} \mathbf{s}_i. \tag{24}$$

From Fig. 3, GLRT-R achieves the optimum detection performance since the FC would receive the observations with no information loss. As for GLRT-C, Case 1 would obtain better performance than Case 2 and 3 since the communication constraint of Case 1 is less stringent than Case 2 and 3. Besides, a canonical one-bit hard-decision fusion method, called the "OR" rule, is also shown in Fig. 3 for comparison. GLRT-C in three cases performs better than the "OR" rule since GLRT-C retains the raw information above the local threshold. Furthermore, the communication bandwidth needed by the GLRT-C can be much less than the "OR" rule since the GLRT-C potentially utilizes the fact that targets only occupy a few range cells in the surveillance space, which leads to a reduction of the communication rate in statistical sense for a scanning cycle. Unlike GLRT-C, "OR" rule measures the communication bandwidth for an individual transmission.

Conclusion

In this study, we address the issue of designing a censoring scheme based on the GLRT in multistatic radar such that communication constraints are satisfied. Each SDC calculates and transmits the local generalized likelihood ratio of multi-dimensional signal only if the local generalized likelihood ratio exceeds a local threshold, where the local threshold controls the communication rate. The fusion rule is derived based on the GLRT-based censoring scheme where each receiver possesses a multi-channel array. The closed-form expression of global PFA is derived given the local thresholds. Numerical results show that the detection performance of GLRT-C approaches that of GLRT-R as the communication constraints become less stringent. Meanwhile, the GLRT-C outperforms the "OR" rule in data compression while achieving better performance.

References

[1] V. Chernyak, Fundamentals of multisite radar system: multistatic radars and multiradar systems, Gordon & Breach Science Publishers, 1998.

[2] E. Fishler, A. Haimovich, R. Blum, ,et al., "Spatial diversity in radars—models and detection performance", IEEE Trans. Signal Process., 2006, 54, (3), pp. 823–838.

[3] P. Varshney, "Distributed detection and data fusion", 1st ed., New York: Springer, 1997.

[4] C. Altay, H. Delic, "Optimal quantization intervals in distributed detection", IEEE Trans. Aerosp. Electron. Syst., 2016, 52, (1), pp. 38-48.

[5] F. Gao, L. Guo, H. Li, et al., "Quantizer design for distributed GLRT detection of weak signal in wireless sensor networks", IEEE Trans. Wirel. Commun., 2015, 14, (4), pp. 2032-2042.

[6] H., Deng, "Polyphase code design for orthogonal netted radar systems", IEEE Trans. Signal Process., 2004, 52, (11), pp. 3126-3135

[7] E. Kelly, "An adaptive deteciton algorithm", IEEE Trans. Aerosp. Electron. Syst., 1986, 22, (1), pp. 115-127.

[8] D. Bruyere, N. Goodman, "Adaptive detection and diversity order in multistatic radar", IEEE Trans. Aerosp. Electron. Syst., 2008, 44, (4), pp. 1615-1623.

[9] S. Zhou, H. L, "Signal fusion-based target detection algorithm for spatial diversity radar", IET radar, sonar & navigation, 2011, 5, (3), pp. 204-214.

[10] J. Liu, Z. Zhang, Y. Cao, et al., "A closed-form expression for false alarm rate of adaptive MIMO-GLRT detector with distributed MIMO radar", Signal Processing, 2013, 93, pp. 2771-2776.

[11] C. Rago, P. Willett, Y. Bar-shalom: "Censoring sensors: a low-communication-rate scheme for distributed detection", IEEE Trans. Aerosp. Electron. Syst., 1996, 32, (2), pp. 554-568

[12] D. Cao, S. Zhou, H. Liu, et al., "Signal censoring and fusing with system-level communication constraints in multistatic radar: a J-divergence and Bhattacharyya distance-based approach", IET Radar, Sonar & Navigation, 2017, 11, (12), pp. 1802-1814.