Adaptive Detection with Censored Data in Multistatic Radar

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Abstract—In multistatic radar, there always exist communication constraints between local radar sites and a fusion center (FC). In this paper, a censored data-based decentralized detection approach is proposed by deriving generalized likelihood ratio test (GLRT) to decrease the need for communication bandwidth. A coherent multi-channel array is utilized at each receiver where only local test statistics (LTSs) exceeding a local threshold are transmitted. The FC makes a global decision through noncoherent integration of local decisions, where the closed-expression for the probability of false alarm (PFA) is derived given communication constraints. Theoretical results are confirmed with Monte-Carlo simulations; Meanwhile, it is demonstrated that the proposed method would maintain better performance than “OR” rule while saving more bandwidth.

Keywords—multistatic radar; censored data; generalized likelihood ratio;

I. INTRODUCTION

Multistatic radar, also termed as multisite radar system or distributed radar, has been a hot topic since radar was invented [1]. Inspired by the concept of multiple-input multiple-output (MIMO) in communication, the distributed MIMO radar has also attracted intensive attentions in recent years. In [2], it can be treated as a multistatic system with widely separated antennas, which again promotes the development of data fusion. Due to the diversity gain, it shows advantages over traditional monostatic radar in many aspects, e.g. target detection and parameter estimation. As a result, one of the vital problems in multistatic radar is to efficiently fuse local observations for global decision making.

Distributed detection can be categorized into two cases [3]: one is to utilize raw observations for data fusion; the other is to make data compression before combining local decisions. Due to no information loss, the former method, also called the centralized fusion, has a potential to achieve the optimum detection performance in some criterion, e.g. Neyman-Pearson criterion. The latter one is termed as the decentralized detection, which alleviates the need for communication bandwidth. As the development of communication, it is convenient to relax communication constraints using wire transport technologies, e.g. fiber optic technique. However, it is still important to complete data fusion with low communication rate for improving the flexibility in deployment of radar sites. Decentralized detection with quantized decisions is one of the most canonical technologies. To some extreme cases, local radar site would transmit one-bit hard-decision to the FC for further fusion [3]. However, such method would bear enormous information loss since the raw observation is quantized into only two levels. In order to improve the detection performance, multi-level quantization has been considered in the case where transmission channels would have greater transmission capacity [4-5].

Many data compression technologies are at the background of communication. In the area of multistatic radar, it is always neglected that targets only occupy a fraction of range cells in surveillance space, which is true for some kind of radar, e.g. ground-to-air radar. Most communication bandwidth is wasted on transmitting noise. Hence, frequent communication between local radar sites and a FC is not necessary. Inspired by the difference of statistical characteristics between null ($H_0$) and alternative ($H_1$) hypotheses, a local threshold can be set at the local radar sites to prevent transmitting local test statistics (LTSs) of very small values, where the communication constraints can be satisfied through adjusting local thresholds.

In this paper, we consider a mutistatic radar system where each receiver possesses a coherent multi-channel array. Each local radar site calculates a LTS using the generalized likelihood ratio test (GLRT), and transmits only when the LTS exceeds a given local threshold. Under independence assumption, the FC receives censored data, and then makes a global decision through noncoherent integration of local decisions. The closed-form expression is derived for global probability of false alarm (PFA).

II. SIGNAL MODEL

In this section, a signal model of multistatic radar is presented with widely separated radar sites. Suppose that a multistatic radar consists of $N_t$ transmitters and $N_r$ receivers. Hence, $N = N_t N_r$, transmit-receive paths, called the spatial diversity channels (SDCs), are obtained at the receive end by designing orthogonal waveforms [6]. Such orthogonality may be imposed in frequency domain or time domain. In the context

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of this paper, it is assumed that the ideal orthogonality among different transmit signals is maintained even for different mutual delays such that

$$\frac{1}{E}\int s_m(t) s_n^\dagger(t+\tau) dt = \begin{cases} 1 & v = w, \tau = 0, \\ 0 & \text{otherwise}, \end{cases}$$

(1)

where $s_m(t)$ is the $m$th transmitter’s transmit signals for $m = 1, 2, \cdots, N_t$, $E$ is energy of transmit signals, and superscript $\dagger$ stands for conjugate transpose. Further suppose that the independent observations can be realized by the geographical separation of the receivers. As a result, $N$ independent observations are obtained at the FC.

In real application, it is common that each receiver possesses a multi-channel system with $N_{rc}$ receive antennas where $i = (n-1)N_r + m$ is an index of SDCs for $n = 1, 2, \cdots, N_r$. Through slow-time sampling, the radar waveform is a simple burst of identical pulses, say $N_{pl}$ in number. This results in a total of $Q = N_{rc}N_{pl}$ complex pulses for an individual range gate. The test data (primary data) under $H_0$ or $H_1$ can be expressed as

$$\begin{align*}
H_0: z_i &= w_i \\
H_1: z_i &= b_i s_i + w_i
\end{align*}$$

(2)

where

• The noise vector $w_i$ corrupting the $i$th SDC’s data is assumed to be circularly symmetric complex Gaussian distributed, i.e., $w_i \sim \mathcal{CN}(0, M_i^c)$, where $\mathcal{CN}(\cdot)$ stands for circularly symmetric, complex Gaussian distribution, and $M_i^c$ is a positive definite covariance matrix of dimension $Q$.

• $s_i$ is a space-time steering vector relative to the specific propagation geometry of the $i$th SDC.

• $b_i$ is a deterministic but unknown complex amplitude of target echo.

The data from reference cells (secondary data), which are free of target component, are usually assumed to be independent and identical distributed (i.i.d.). Suppose that the number of reference cells corresponding to the $i$th SDC is $K_i$. The data of the $j$th reference cell, denoted by $z_{i,j}$ for $j = 1, 2, \cdots, K_i$, also follow the circularly symmetric complex Gaussian distribution, i.e., $z_{i,j} \sim \mathcal{CN}(0, M_i^c)$ . All the measurements are hence governed by

$$\begin{align*}
H_0: & \quad \begin{cases} z_i & \sim \mathcal{CN}(0, M_i^c) \\
\mathbf{z}_{i,j} & \sim \mathcal{CN}(0, M_i^c) \end{cases} \\
H_1: & \quad \begin{cases} z_i & \sim \mathcal{CN}(b_i s_i, M_i^c) \\
\mathbf{z}_{i,j} & \sim \mathcal{CN}(0, M_i^c) \end{cases}
\end{align*}$$

(3)

The probability density function (PDF) under $H_k$ for $k = 0, 1$ can be denoted by

$$f(z_i|H_k) = \frac{1}{\pi^Q |M_k^c|} \exp\left( (z_i - kb_i s_i) M_k^{-1} (z_i - kb_i s_i) \right)$$

$$f(z_{i,j}|H_k) = \frac{1}{\pi^Q |M_k^c|} \exp\left( z_{i,j} M_k^{-1} z_{i,j} \right)$$

(4)

with $|M|$ denoting the determinant of a matrix.

III. ADAPTIVE DETECTION WITH CENSORED DATA

Although the Neyman-Pearson criterion achieves the optimum performance, it is difficult in real application to obtain some parameter under $H_1$, such as signal-to-noise (SNR). A common and suboptimum method is to utilize GLRT at the FC where the fusion detector is derived by substituting unknown parameters with their maximum likelihood estimates [7]. Let $Z_i$ be a matrix that comprises all measurements received by the $i$th SDC such that $Z_i = [z_i, z_{i,1}, \cdots, z_{i,K_i}]$. In centralized system, the $Z_i$s are transmitted directly to the FC for global decision. Under independence assumption, the GLRT with raw observations (GLRT-R) for a multistatic system can be denoted by [8-10]

$$T_g = \log \frac{ \max_{\{b_i, M_i^c\} = \{1, 2, \cdots, K_i\}} f(Z_i|H_1) } { \max_{\{b_i, M_i^c\} = \{1, 2, \cdots, K_i\}} f(Z_i|H_0) }$$

$$= \sum_{i=1}^N \log \frac{ \max_{\{b_i, M_i^c\} = \{1, 2, \cdots, K_i\}} f(Z_i|H_1) } { \max_{\{b_i, M_i^c\} = \{1, 2, \cdots, K_i\}} f(Z_i|H_0) } = \sum_{i=1}^N x_i \geq \eta$$

(5)

where $\eta$ is a global threshold corresponding to a given PFA, and $x_i$ is the LTS from the $i$th SDC. As shown in [7], $x_i$ can be written as

$$x_i = (K_i + 1) \log \left( \frac{1 + z_i^\dagger \tilde{M}_i^{-1} z_i}{1 + z_i^\dagger \tilde{M}_i^{-1} S_i \tilde{M}_i^{-1} z_i} \right)$$

(6)

where $\tilde{M}_i = \sum_{j=1}^{K_i} z_{i,j} z_{i,j}^\dagger$ involves only the secondary data which follows the well-known Wishart distribution. A condition that we impose here is $K_i \geq Q$, such that the matrix $\tilde{M}_i$ is nonsingular with probability one. From (5), both the denominator and numerator can be maximized individually for each SDC.

In some scenario, the communication bandwidth between local radar sites and the FC may not afford transmitting raw observations or LTSs. It is an efficient method to utilize censored data to alleviate the demand for communication bandwidth where only the most “informative” data are transmitted [11-12]. Assume that targets dominate only a fraction of range cells in surveillance space. Most of the communication bandwidth is wasted on transmitting just noise. Censoring scheme may decrease the communication load while guaranteeing a certain level of performance. By virtue of the
difference between the statistical characteristics of the observations under $H_0$ and $H_1$, the core of such method is to utilize a local threshold to discard some “uninformative” LTSs. Therefore, the local communication rate can be denoted by [10]

$$\alpha_i = \Pr\{x_i > \eta_i \mid H_0\} \tag{7}$$

where $\alpha_i \in [0,1]$ is a constraint on communications per resolution cell, and $\eta_i$ is the local threshold for the $i$th SDC. For example, we consider a scenario where the sampling frequency is $f_s = 2$ MHz, the maximum detection range is $R_{\text{max}} = 450$ km, and the pulse repetition interval is $T_p = 3$ ms. Suppose that a faithful rendition of local observation needs $q = 12$ bits. For centralized detection, the communication rate requires at least

$$Q = \frac{2R_{\text{max}}f_sq}{cT_p} = 2.86 \text{ MB/s.} \tag{8}$$

where $c$ is a constant denoting the light velocity. Given $\alpha_{\text{tot}} = 0.01$, the communication constraint decreases to $\alpha_{\text{tot}}Q = 29.29 \text{ kB/s}$, where the local threshold $\eta_i$ controls the communication rate such that only $\alpha_i \times 100\%$ of all data are transmitted in a scanning cycle. Hence, the local decision rule can be written as

$$u_i = \begin{cases} x_i, & x_i > \eta_i \\ q_i, & x_i < \eta_i \end{cases} \tag{9}$$

where $q_i$ is a discrete decision for $x_i \in [0,\eta_i)$ . It is easy to verify that we have $q_i = 0$ based on GLRT [5]. From (9), the LTS $u_i$ has both continuous and point-mass characteristics. Considering randomization decision [12], the global decision rule can be written as

$$z = \sum_{i=1}^N u_i \quad \begin{cases} \geq \eta, & \text{decide } H_1 \\ < \eta, & \text{decide } H_0 \\ = \eta, & \text{decide } H_i \text{ with probability } \gamma \end{cases} \tag{10}$$

where $\eta$ is a global decision threshold to control the PFA, $\gamma \in [0,1]$ is a randomization parameter to satisfy given PFA if possible, $z$ is the global test statistic. The topology of GRLT with censored data (GLRT-C) is shown in Fig.1.

![Fig. 1. Topology of GLRT-C](image)

IV. FALSE ALARM RATE

For a multistatic system with $N$ SDCs, the global PDFs under $H_0$ can be derived by $N$-fold convolution, which can be written in the form of

$$f_0(z) = g_0(z)e(z - \eta_{\text{min}}) + \prod_{i=1}^N (1 - \alpha_i) \delta(z) \tag{11}$$

where $g_0(z)$ is a deterministic but unknown function, $f_0(z)$ is the PDF of global test statistic, and $\eta_{\text{min}}$ can be written as

$$\eta_{\text{min}} = \min_{i=1,2,\cdots,N} \eta_i \tag{12}$$

By virtue of randomization decision rule, the PFA $P_0$ can be derived as

$$P_0 = \Pr\{z \geq \eta \mid H_0\} = \int_{\eta}^{\infty} f_0(z) \, dz = \frac{\eta}{P_{\text{tot}}} \tag{13}$$

Due to the point-mass characteristics, multistatic radar system may not satisfy any given PFA just through adjusting global threshold. The relationship between PFA and the global threshold can be categorized into two cases. Case 1: $\eta > 0$. From (11), the global test statistic has continuous characteristics for $z \in [\eta, +\infty)$, which leads to

$$\Pr\{z = \eta \mid H_0\} = 0. \text{ Therefore, (13) can be rewritten as} \tag{14}$$

$$P_0 = \Pr\{z \geq \eta \mid H_0\} = \int_{\eta}^{\infty} g_0(z) \, dz,$$

Despite the difficulty in calculating $g_0(z)$ directly, the conditional probability is much easier to derive given all SDCs’ transmission states. Define $S_j$ to represent one of all SDCs’ transmission states for $j = \sum_{i=1}^N b_i 2^{i-1}$, where $b_i = 1$ if the $i$th SDC transmits, and $b_i = 0$ otherwise. The PFA can be rewritten as a sum version of conditional probabilities, i.e.,

$$P_0 = \sum_{j=1}^{2^N-1} P_0(S_j) = \int_{\eta}^{\infty} g_0(z) \, dz = \sum_{j=1}^{2^N-1} P_0(S_j) P_0(S_j) \tag{15}$$

where $P_0(S_j)$ denotes the occurrence probability of event $S_j$, $g_0(z \mid S_j)$ is the conditional PDF given $S_j$, and $P_0$ is the conditional PFA. As for the local test statistic $x_i$ in (6), Kelly [7] has verified that it follows the exponential distribution with mean $(K_i + 1)/(K_i + 1 - Q_i)$ under $H_0$, i.e.,

$$f(x_i \mid H_0) = \frac{K_i + 1 - Q_i}{K_i + 1} \exp\left[-\frac{(K_i + 1 - Q_i) x_i}{K_i + 1}\right], \tag{16}$$

Hence, $\alpha_i$ can be written as

$$\alpha_i = \exp\left[-\frac{(K_i + 1 - Q_i) x_i}{K_i + 1}\right] \tag{17}$$
Moreover, $P_0(S_j)$ and $P'_0$ can be written as

$$P_0(S_j) = \prod_{i,j \neq j} \alpha_i \prod_{i,j \neq j} (1-\alpha_i)$$

(18)

and

$$P'_0 = \Pr \left( \sum_{i,j > 0} x_i > \eta \mid S_j, H_0 \right),$$

(19)

respectively. The conditional PDF of $x_i$ in (19) can be denoted by

$$f_i(x_i \mid S_j) = \frac{f_i(x_i)}{\alpha_i} \exp \left[ -\frac{(K_i + 1 - Q_i)(x_i - \eta)}{K_i + 1} \right] \exp \left( x_i - \eta \right).$$

(20)

which still follows the exponential distribution. Therefore, (19) can be simplified to

$$P'_0 = \Pr \left( \sum_{i,j \neq j} (x_i - \eta) > \eta - \sum_{i,j \neq j} \eta \mid S_j, H_0 \right)$$

$$= 1 - F_{\text{sum}} \left( \eta - \sum_{i,j \neq j} \eta \right),$$

(21)

where $F_{\text{sum}}$ denotes the cumulative distribution function (CDF) of a sum of exponential variates governed by (16). Especially, $F_{\text{sum}}(\cdot)$ is the CDF of the gamma distribution when the number of secondary data and the signal vector length are the same among all SDCs.

Case 2: $\eta = 0$. In this case, we have $\Pr(z = \eta \mid H_0) > 0$.

Therefore, the PFA denoted by (13) can be simplified to

$$P_0 = \int_{-\infty}^{\infty} f_0(z)dz + \gamma \Pr(z = 0 \mid H_0)$$

$$= 1 - (1 - \gamma)P_0(S_0),$$

(22)

where $P_0(S_0)$ can be denoted by

$$P_0(S_0) = \prod_{i=1}^{N} (1-\alpha_i).$$

(23)

As for a given PFA larger than $P_0(S_0)$, decreasing the global threshold cannot satisfy the desired PFA anymore. However, censoring scheme may achieve any PFA through setting an appropriate randomization parameter $\gamma$.

It should be noticed that the detection performance can be analyzed in the same method following (13)-(21). However, it is not easy to achieve the closed-form expression of $F_{\text{sum}}$ under $H_1$ currently, which needs further investigation.

V. SIMULATION RESULTS

In this section, numerical results are conducted to verify the effectiveness of the proposed method. For simplicity, we consider a multistatic system with two transmitters and two receivers. As a result, there are four SDCs for data fusion. Suppose that all SDCs have the same vector length where $Q_1 = Q_2 = Q_3 = Q_4 = 10$, and the numbers of secondary data are $K_1 = K_2 = K_3 = K_4 = 20$.

In order to examine the PFA of proposed detector, we consider three cases with different local communication rates.

Case 1: $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 0.05$, $\alpha_4 = 0.05$, i.e., the first receiver not only receives signals, but also acts as a FC.

Case 2: $\alpha_1 = 0.01$, $\alpha_2 = 0.01$, $\alpha_3 = 0.01$, $\alpha_4 = 0.01$, i.e., all communication constraints are identical.

Case 3: $\alpha_1 = 0.002$, $\alpha_2 = 0.004$, $\alpha_3 = 0.006$, $\alpha_4 = 0.008$, i.e., communication constraints are distinct among all SDCs.

The PFA as a function of global threshold for three cases is reported in Fig. 2, where the dashed, solid and dashed-dotted lines indicate the results obtained from (13)-(21) for Case 1, 2, and 3, respectively. As for Case 1, the global test statistic has only continuous characteristics. It can achieve any PFA through adjusting global threshold. For Case 2 and 3, the global test statistics have both continuous and point-mass characteristics. Although randomization decision may achieve better detection performance, we preferred to set $\gamma = 0$ in real application since randomization decision occurs only when the FC receives no data. In this scenario, $H_1$ is chose randomly with probability $\gamma$, which makes a decision with no further information. Hence, horizontal lines are plotted when the PFA is larger than $P_0(S_0)$. The symbols represent the corresponding results obtained from the Monte-Carlo simulations. The number of independent trials is $10^5$. It is shown that the theoretical results match the Monte-Carlo simulations pretty well.

The probabilities of detection (PDs) of three cases are shown in Fig. 3. No antenna alignment errors are considered here. Therefore, target steering vectors were assumed to be accurate. Without loss of generality, assume that the output SNRs $\alpha_i$s are the same among all SDCs, which can be denoted by

![Fig. 2. PFA versus global thresholds in three cases.](image-url)
\[ a_i = |b_i|^2 s_i' M_i^{-1} s_i. \]  

(24)

From Fig. 3, GLRT-R achieves the optimum detection performance since the FC would receive the observations with no information loss. As for GLRT-C, Case 1 would obtain better performance than Case 2 and 3 since the communication constraint of Case 1 is less stringent than Case 2 and 3. Besides, a canonical one-bit hard-decision fusion method, called the “OR” rule, is also shown in Fig. 3 for comparison. GLRT-C in three cases performs better than the “OR” rule since GLRT-C retains the raw information above the local threshold. Furthermore, the communication bandwidth needed by the GLRT-C can be much less than the “OR” rule since GLRT-C potentially utilizes the fact that targets only occupy a few range cells in the surveillance space, which leads to a reduction of the communication rate in statistical sense for a scanning cycle. Unlike GLRT-C, “OR” rule measures the communication bandwidth for an individual transmission.

CONCLUSION

In this study, we address the issue of designing a censoring scheme based on the GLRT in multistatic radar such that communication constraints are satisfied. Each SDC calculates and transmits the local generalized likelihood ratio of multi-dimensional signal only if the local generalized likelihood ratio exceeds a local threshold, where the local threshold controls the communication rate. The fusion rule is derived based on the GLRT-based censoring scheme where each receiver possesses a multi-channel array. The closed-form expression of global PFA is derived given the local thresholds. Numerical results show that the detection performance of GLRT-C approaches that of GLRT-R as the communication constraints become less stringent. Meanwhile, the GLRT-C outperforms the “OR” rule in data compression while achieving better performance.

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