Prior Knowledge Based Transmit Nulling Method for MIMO Radar

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***Abstract*—In this paper, a multiple-input-multiple-output (MIMO) radar transmit nulling method is proposed in the presence of fast moving interference. The MIMO radar transmit waveform design is divided into two steps: transmit waveform covariance matrix optimization and transmit waveform synthesis to approximate the optimized covariance matrix. In the first step, the prior knowledge of interference is utilized to optimize covariance matrix and the derivative constraint is introduced to widen nulls. In the second step, most existing methods, e.g. cyclic algorithm (CA) and sequential iterative algorithm (SIA), synthesize the waveform with constant modulus under least-squares criterion. As the value at the null even can be neglected in the calculation of least-squares, these methods cannot form deep enough nulls at the interference direction. Aiming at this problem, a block coordinate descent (BCD) method is proposed to optimize the initial waveform synthesized by CA. Besides, the optimized waveform is still constant modulus. Numerical results show the efficiency of the proposed method.**

Keywords—MIMO radar; transmit nulling; interference; cyclic algorithm; block coordinate descent; constant modulus waveform

I. INTRODUCTION

Each transmit antenna of multiple-input-multiple-output (MIMO) radar can freely choose the transmit waveform, which provides MIMO radar with the advantage of waveform diversity. In general, MIMO radar system can be classified into two categories according to the configuration of antennas: MIMO radar with widely separated antennas and MIMO radar with colocated antennas. In this paper, we focus on the colocated MIMO radar, in which transmitters and receivers are close so that all transmitters see the same target radar cross section and this provides more flexibility in the beampattern design [1].

The waveform design for MIMO radar beampattern formulation has recently received the considerable attention in the literature [2–11]. In summary, there are two ways to obtain the waveform for desired transmit beampattern. Since MIMO radar transmit beampattern is characterized by the covariance matrix, the first one considers the two-step process by focusing on the optimization of the waveform covariance matrix and waveform synthesis to approximate the optimized covariance matrix under the practical constraints. For the optimization of covariance matrix, a semi-definite quadratic programming algorithm was presented in [2], where both the design of the beampattern matching and the minimum sidelobe beampattern were taken into account to optimize the waveform covariance matrix. In [3], the constrained optimization problem about waveform covariance matrix was converted into an unconstrained optimization problem. For waveform synthesis problem, the waveform is usually required to satisfy the constant modulus constraint (CMC) so that the radio frequency amplifiers of the antennas have maximum power efficiency. In [4], the cyclic algorithm (CA) was introduced to synthesize the waveform with CMC from an infinite alphabet. In [5], an algorithm that generates binary-phase shift keying waveform to approximate the given beampattern was proposed. However, this algorithm just can apply to the symmetric beampattern or the shift of symmetric beampattern. In [6], a sequential iterative algorithm (SIA) to synthesize the transmit waveform enforcing the CMC was proposed.

The second way is to design the waveform directly without covariance matrix optimization. In [7], Benjamin Friedlander obtained transmit waveform by designing rank-1 beamformers. In [8], two methods of designing weight matrice for a set of finite-alphabet waveform were proposed. In [9], discrete Fourier transform coefficients were chosen to represent the region of interest and then transmit waveform was obtained by a combination of a set of orthogonal waveform. In [10], in the presence of signal-dependent interference, both the transmit waveform and receive filter are designed to maximize the signal to interference plus noise ratio. In [11], the joint design problem of the space-time transmit waveform and the space-time receive filter for a moving target is considered.

Because of the advantage of waveform diversity, compared with the single-input-multiple-output (SIMO) radar, MIMO radar can realize simultaneous multibeam and transmit nulling in one pulse by designing waveform. Therefore, in this paper, we investigate the MIMO radar transmit nulling with simultaneous multibeam in the presence of fast moving interference. A convex problem is obtained to design the transmit waveform covariance matrix with low sidelobe and wide nulls. For waveform synthesis, because existing methods are unable to form deep enough nulls at the interference direction, we propose a block coordinate descent (BCD) method to optimize the initial transmit waveform synthesized by CA. After the BCD process, the optimized waveform still

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satisfies the CMC. Besides, because we regard every row of waveform matrix as a block of variables in BCD, the block number of the optimization problem is small and the optimization process is fast.

The rest sections of this paper is organized as follows. In section II, the signal model and the null constraint are introduced and then a convex model is proposed to obtain the optimal waveform covariance matrix. In section III, a BCD method is proposed to optimize the initial transmit waveform from CA. In section IV, some simulation results are provided to demonstrate the superiorities of the designed waveform. Finally, in section V, we conclude the paper.

Notation: Boldface upper case letters denote matrices, boldface lower case letters denote vectors and italics denote scalars. $\mathrm{Re}(\bullet)$ denotes the real part. $angle(\bullet)$ denotes the phase of a complex number. The superscripts $(\bullet)^{\mathrm{T}}$, $(\bullet)^{*}$ and $(\bullet)^{\mathrm{H}}$ denote transpose, complex conjugate and conjugate transpose, respectively. $\odot$ denotes the Hadamard product. $tr(\bullet)$ denotes the trace of a matrix. $diag(\boldsymbol{x})$ is a diagonal matrix formed with $\boldsymbol{x}$ as its principal diagonal. $\|\bullet\|_2$ denotes the Euclidean norm of a vector. i is the imaginary unit.

II. Waveform Covariance Matrix Optimization

Consider a colocated MIMO radar consisting of M transmit antennas. Each antenna radiates a continuous phase encoding signal with L signal samples. The transmit waveform matrix can be written as

$$\mathbf{X} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_M \end{bmatrix} \tag{1}$$

where $\boldsymbol{x}_m = \left[x_m(1), x_m(2), \cdots, x_m(L)\right]$ is a row vector, which represents the waveform transmitted by the m-th antenna. Then the waveform covariance matrix can be written as

$$\mathbf{R} = \frac{1}{L}\mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{2}$$

The received signal power at azimuth angle θ can be represented as

$$P(\theta) = \boldsymbol{a}^{\mathrm{H}}(\theta)\mathbf{R}\boldsymbol{a}(\theta) \tag{3}$$

where the transmit steering vector is given by

$$\boldsymbol{a}(\theta) = \left[e^{\mathrm{i}2\pi z_1 \sin\theta/\lambda}, e^{\mathrm{i}2\pi z_2 \sin\theta/\lambda}, \cdots, e^{\mathrm{i}2\pi z_M \sin\theta/\lambda}\right]^{\mathrm{T}}$$

z_m represents the location of antenna and λ is the transmission wavelength.

The interference's azimuth angle θ_k and the number of interference K are the prior knowledge. For the passive interference e.g. aluminum chaff, these prior knowledge can be obtained by the parameter estimation method in [15] and [16]. To form nulls at interference's direction, the waveform covariance matrix should satisfy the following constraint

$$tr\left(\mathbf{V}^{\mathrm{H}}\mathbf{R}\mathbf{V}\right) \le \rho \tag{4}$$

where $\mathbf{V} = \left[a(\theta_1), a(\theta_2), \cdots, a(\theta_K)\right]$ represents interference subspace. ρ is a parameter set in advance, which decides the orthogonality between interference subspace and transmit waveform. ρ can be roughly regarded as the tolerable maximal power that radiates the interference.

Constraint (4) only can form sharp nulls on the beampattern, so the fast moving interference may escape the null in a short time. For this reason, the derivative constraint in [12] is used to extend the interference subspace and widen the nulls

$$tr\left(\mathbf{U}^{\mathrm{H}}\mathbf{R}\mathbf{U}\right) \le \rho \tag{5}$$

where

$$\mathbf{U} = \left[\mathbf{B}^0\mathbf{V}, \cdots, \mathbf{B}^p\mathbf{V}\right]$$

$$\mathbf{B}^r = \left(\sum_{m=1}^{M} z_m^{2r}\right)^{-1/2} diag\left(\left[z_1^r, z_2^r, \cdots, z_M^r\right]\right),\ r = 1, \cdots, p$$

p represents the order of the derivative constraint. $\mathbf{B}^0$ is defined as an identity matrix, so when the zero-order derivative constraint is used, the derivative constraint is same as (4).

Finally, to obtain the beampattern with the desired nulls and the low sidelobe, the following problem model is used to optimize the waveform covariance matrix

$$\begin{aligned} \min_{R}\ \max_{\theta_s \in \Omega_{side}}\quad & \boldsymbol{a}^{\mathrm{H}}(\theta_s)\mathbf{R}\boldsymbol{a}(\theta_s) \\ s.t.\quad & \boldsymbol{a}^{\mathrm{H}}(\theta_i)\mathbf{R}\boldsymbol{a}(\theta_i) \ge P_i,\ i = 1, \cdots, I \\ & tr\left(\mathbf{U}^{\mathrm{H}}\mathbf{R}\mathbf{U}\right) \le \rho \\ & R_{mm} = c/M,\ m = 1, \cdots, M \\ & \mathbf{R} \ge 0 \end{aligned} \tag{6}$$

where, Ω_{side} represents the sidelobe region, P_i is the desired power at the interesting azimuth angle θ_i. R_{mm} is the m-th diagonal elements of the matrix $\mathbf{R}$. c is the total transmit power. Above problem is convex [13], the optimal $\mathbf{R}$ can be obtained easily by using the convex optimization toolbox CVX in MATLAB.

III. Optimize Waveform via BCD

According to the optimal $\mathbf{R}$, we first use CA to obtain the initial waveform matrix $\bar{\mathbf{X}}$. The details of CA are referred to [4]. Because CA approximates the optimal $\mathbf{R}$ based on the least-squares criterion, the small residuals are put on very small weight [13], that leads the designed waveform matrix by CA can't form deep enough nulls.

In order to form the desired nulls at the interference's direction, a BCD method is proposed to optimize the initial waveform matrix $\bar{\mathbf{X}}$ row by row to fulfill constraint (5). A proof of the convergence for the BCD method can be found in [14]. For the waveform matrix, constraint (5) can be rewritten as

$$tr\left(\mathbf{U}^{\mathrm{H}}\mathbf{X}\mathbf{X}^{\mathrm{H}}\mathbf{U}\right)\leq L\rho \tag{7}$$

$\bar{\mathbf{X}}$ is divided into M blocks as (1):

$$\bar{\mathbf{X}}=\begin{bmatrix}\bar{\boldsymbol{x}}_1\\ \bar{\boldsymbol{x}}_2\\ \vdots\\ \bar{\boldsymbol{x}}_M\end{bmatrix}$$

Suppose that the *j*-th block variable is optimized at present, the following problem model can be obtained

$$\begin{aligned}&\min_{\boldsymbol{\omega}_j}\ tr\left(\mathbf{U}^{\mathrm{H}}\hat{\mathbf{X}}\hat{\mathbf{X}}^{\mathrm{H}}\mathbf{U}\right)\\ &s.t.\ \ \left|\boldsymbol{\omega}_j(l)\right|=1,\ l=1,\cdots,L\end{aligned} \tag{8}$$

where

$$\hat{\mathbf{X}}=\begin{bmatrix}\bar{\boldsymbol{x}}_1\\ \vdots\\ \bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\\ \vdots\\ \bar{\boldsymbol{x}}_M\end{bmatrix}$$

$\boldsymbol{\omega}_j$ is a $1\times L$ vector which represents the phase change of the *j*-th block variable so that $\hat{\mathbf{X}}$ still satisfies the CMC.

Let u_{mq} represent the (*m*-th, *q*-th) element of $\mathbf{U}$. The objective function of (8) can be written as

$$\begin{aligned}tr\left(\mathbf{U}^{\mathrm{H}}\hat{\mathbf{X}}\hat{\mathbf{X}}^{\mathrm{H}}\mathbf{U}\right)=\sum_{q=1}^{K(p+1)}\Bigg[&\sum_{\substack{m=1\\m\neq j}}^{M}\left|u_{mq}\right|^2\left\|\bar{\boldsymbol{x}}_m\right\|_2^2\\ &+\left|u_{jq}\right|^2\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)^{\mathrm{H}}\\ &+\sum_{\substack{n=1\\n\neq j}}^{M}u_{jq}^{*}u_{nq}\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)\bar{\boldsymbol{x}}_n^{\mathrm{H}}\\ &+\sum_{\substack{m=1\\m\neq j}}^{M}u_{jq}u_{mq}^{*}\bar{\boldsymbol{x}}_m\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)^{\mathrm{H}}\\ &+\sum_{\substack{m=1\\m\neq j}}^{M}\sum_{\substack{n=1\\n\neq j\\n\neq m}}^{M}u_{mq}^{*}u_{nq}\bar{\boldsymbol{x}}_m\bar{\boldsymbol{x}}_n^{\mathrm{H}}\Bigg]\end{aligned} \tag{9}$$

where K is the number of interference and p is the order of the derivative constraint.

It is obvious that the first item and the last item in (9) are constant and because $\boldsymbol{\omega}_j$ only changes the phase of $\bar{\boldsymbol{x}}_j$, the second item in (9) is also constant. Ignoring the constant items in (9), problem (8) can be written as

$$\begin{aligned}&\min_{\boldsymbol{\omega}_j}\ 2\sum_{q=1}^{K(p+1)}\sum_{\substack{m=1\\m\neq j}}^{M}\mathrm{Re}\left(u_{jq}u_{mq}^{*}\bar{\boldsymbol{x}}_m\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)^{\mathrm{H}}\right)\\ &s.t.\ \ \left|\boldsymbol{\omega}_j(l)\right|=1,\ l=1,\cdots,L\end{aligned} \tag{10}$$

and because

$$\bar{\boldsymbol{x}}_m\left(\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j\right)^{\mathrm{H}}=\left(\bar{\boldsymbol{x}}_m\odot\bar{\boldsymbol{x}}_j^{*}\right)\boldsymbol{\omega}_j^{\mathrm{H}}$$

(10) can be rewritten as

$$\begin{aligned}&\min_{\boldsymbol{\omega}_j}\ 2\,\mathrm{Re}\left[\left(\sum_{q=1}^{K(p+1)}\sum_{\substack{m=1\\m\neq j}}^{M}u_{jq}u_{mq}^{*}\bar{\boldsymbol{x}}_m\odot\bar{\boldsymbol{x}}_j^{*}\right)\boldsymbol{\omega}_j^{\mathrm{H}}\right]\\ &s.t.\ \ \left|\boldsymbol{\omega}_j(l)\right|=1,\ l=1,\cdots,L\end{aligned} \tag{11}$$

Let $\boldsymbol{u}_j$ represent the *j*-th row of $\mathbf{U}$, the following formula can be obtained

$$\sum_{q=1}^{K(p+1)}\sum_{\substack{m=1\\m\neq j}}^{M}u_{jq}u_{mq}^{*}\bar{\boldsymbol{x}}_m=\boldsymbol{u}_j\mathbf{U}^{\mathrm{H}}\bar{\mathbf{X}}-\left\|\boldsymbol{u}_j\right\|_2^2\bar{\boldsymbol{x}}_j$$

$\boldsymbol{\eta}$ is defined as

$$\boldsymbol{\eta}=\left\|\boldsymbol{u}_j\right\|_2^2\bar{\boldsymbol{x}}_j-\boldsymbol{u}_j\mathbf{U}^{\mathrm{H}}\bar{\mathbf{X}}$$

So, (11) is equivalent to

$$\begin{aligned}&\min_{\boldsymbol{\omega}_j}\ -2\,\mathrm{Re}\left[\left(\boldsymbol{\eta}\odot\bar{\boldsymbol{x}}_j^{*}\right)\boldsymbol{\omega}_j^{\mathrm{H}}\right]\\ &s.t.\ \ \left|\boldsymbol{\omega}_j(l)\right|=1,\ l=1,\cdots,L\end{aligned} \tag{12}$$

The closed-form solution of (12) is

$$\boldsymbol{\omega}_j=\exp\left(\mathrm{i}\bullet angle\left(\boldsymbol{\eta}\odot\bar{\boldsymbol{x}}_j^{*}\right)\right) \tag{13}$$

where i is the imaginary unit.

We replace $\bar{\boldsymbol{x}}_j$ by $\bar{\boldsymbol{x}}_j\odot\boldsymbol{\omega}_j$ and conduct the similar procedure to optimize the remaining blocks in $\bar{\mathbf{X}}$. The above process is repeated until the waveform matrix fulfills (7). Finally, the total iteration procedure of the BCD method is summarized in Algorithm 1. Each iteration of the proposed algorithm need solve M problems, each of which corresponds to the computational complexity of $O(ML)$.

Algorithm 1: CA+BCD Algorithm

Input: Optimal covariance matrix $\mathbf{R}$, the number of antenna M, the length of signal sample L and the parameter ρ;

Output: The waveform matrix $\bar{\mathbf{X}}$ satisfies (7);

1: According to $\mathbf{R}$, use CA to obtain the initial waveform $\bar{\mathbf{X}}$;

2: Set $j=1$;

3: Select the j-th row of $\mathbf{U}$ and $\bar{\mathbf{X}}$, obtain $\boldsymbol{\omega}_j$ through (13);

4: Use $\bar{\boldsymbol{x}}_j \odot \boldsymbol{\omega}_j$ to update the j-th row of $\bar{\mathbf{X}}$;

5: Set $j=j+1$;

6: If $j \le M$, return to step 3; Otherwise turn to next step;

7: If $tr\left(\mathbf{U}^{\mathrm{H}}\bar{\mathbf{X}}\bar{\mathbf{X}}^{\mathrm{H}}\mathbf{U}\right) \le L\rho$, output the waveform matrix $\bar{\mathbf{X}}$; Otherwise return to step 2.

IV. Numerical Results

In this section, we evaluate the performance of the proposed algorithm considering a uniform linear array (ULA) of 16 transmit antennas (i.e., $M=16$) with a halfwavelength interelement spacing. We focus on a scenario where -35°, 0° and 45° are regarded as interesting azimuth angle (i.e., $I=3$) and the interference's azimuth angles are -60° and 20° (i.e., $K=2$). And we assume that the sidelobe region Ω_{side} is as follows:

$$\left[-90^{\circ},-45^{\circ}\right]\cup\left[-25^{\circ},-10^{\circ}\right]\cup\left[10^{\circ},35^{\circ}\right]\cup\left[55^{\circ},90^{\circ}\right]$$

In all simulation experiments, P_i ($i=1,2,3$), c and ρ are set as $0.9M^2/3$, M and $10^{-3.5}$ respectively and the number of signal simple $L=200$. Besides, the stop criterion of CA and SIA refer to [4] and [6] respectively and the stop threshold of CA and SIA are both set to 10^{-5}.

In the first simulation experiment, only the zero order derivative constraint is considered, that is $\mathbf{U}=\mathbf{V}$. The optimal $\mathbf{R}$ is obtained by solving model (6). We compare the proposed algorithm with CA and SIA. The waveform synthesized by CA is used as the initial waveform matrix for BCD. Fig. 1 shows the transmit beampattern of the optimal $\mathbf{R}$ and the transmit beampattern of the waveforms synthesized by CA, SIA and CA+BCD respectively. The transmit beampattern is obtained by (2) and (3).

In the second simulation experiment, the first order derivative constraint is considered, that is $\mathbf{U}=\left[\mathbf{B}^0\mathbf{V},\mathbf{B}^1\mathbf{V}\right]$. Fig. 2 shows the transmit beampattern of the optimal $\mathbf{R}$ and the transmit beampattern of the waveforms synthesized by CA, SIA and CA+BCD in the case of considering first order derivative constraint.

As can be seen in Fig. 1 and Fig. 2, because both CA and SIA approximate the optimal $\mathbf{R}$ based on the least-squares criterion, they can't form deep enough nulls at interference's azimuth angles. From Fig. 1 and Fig. 2, we also can find that after optimizing by BCD, the transmit beampattern has desired nulls at interference's azimuth angles and in Fig. 2, because of the first order derivative constraint, the nulls become wider compared with Fig. 1.

In the third simulation experiment, we show the behavior of the value of $tr\left(\mathbf{U}^{\mathrm{H}}\mathbf{X}\mathbf{X}^{\mathrm{H}}\mathbf{U}\right)$ (hereinafter referred to as null constraint value) versus iteration number in Fig. 3(a) and versus computational time in Fig. 3(b) in the case of considering the first order derivative constraint. To compare these three algorithms more exactly, we still run CA and SIA when they reach the stop threshold 10^{-5} and the maximum iteration number is 700 for both CA and SIA. We choose the waveform matrix which CA output when it reaches the stop threshold 10^{-5} as the initial waveform for BCD.

In Fig. 3(a), for both CA and SIA, the null constraint value decreases fast in early and then is stuck at a local minimum. Additionally, as we can see, the null constraint value is pushed to reach the value of optimal $\mathbf{R}$ because of using BCD. From Fig. 3(b), we can find that because the waveform matrix is divided into M blocks, the BCD process is fast and only takes up little computational time of the whole process.

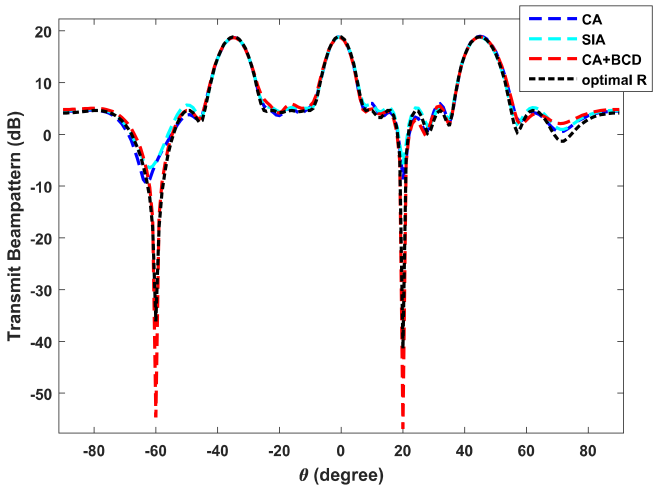

Fig. 1. Transmit beampattern of CA, SIA, CA+BCD and optimal $\mathbf{R}$ considering the zero order derivative constraint.

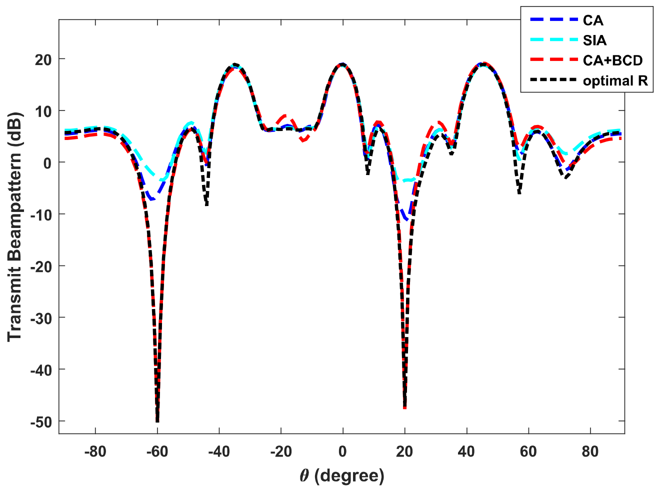

Fig. 2. Transmit beampattern of CA, SIA, CA+BCD and optimal $\mathbf{R}$ considering the first order derivative constraint.

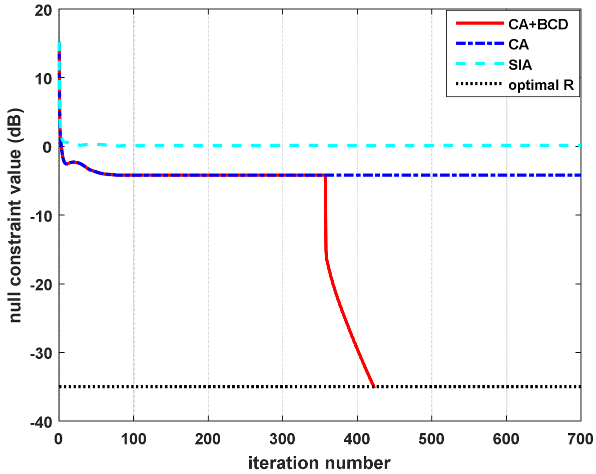

Fig. 3(a). The value of the null constraint versus iteration number.

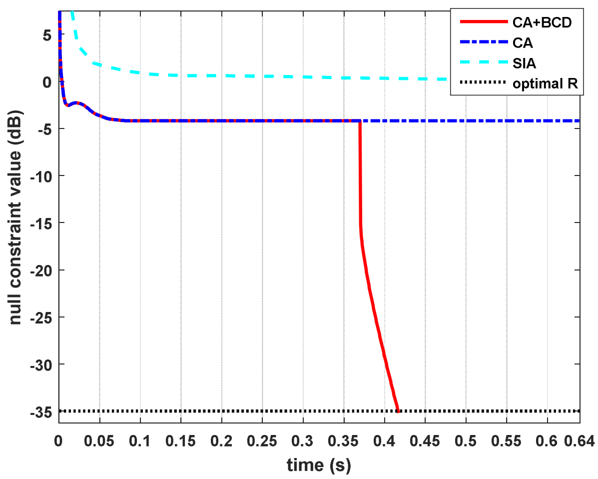

Fig. 3(b). The value of the null constraint versus computational time.

V. Conclusion

In this paper, we propose a transmit nulling method for MIMO radar. To realize transmit nulling, we first set up a convex model to obtain the desired covariance matrix. Then, aiming at the shortcoming of existing waveform synthesis methods, we propose a BCD method to optimize transmit waveform to form desired nulls at interference direction and the waveform optimized by BCD fulfills the constant modulus constraint. The simulation experiments show that the nulls formed by the BCD are closer to the optimal nulls than the nulls formed by CA and SIA. In addition, due to dividing the waveform matrix into M blocks, the process of producing nulls is very fast.

It should be noted that while forming the nulls, the BCD process inevitably affects the mainlobe and the sidelobe of the initial waveform synthesized by CA. In our experiments, it is shown that this method has very little effect on the mainlobe and the sidelobe. The theoretical analysis about the effect of BCD on the mainlobe and the sidelobe needs to be further studied in the future.

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