Online Transmit Beampattern Notching for Colocated MIMO radar

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***Abstract*—Radar system may receive deceptive targets from hostile active Electronic Counter Measurement (ECM) devices if its signal is detected by an ECM device. In practice, once the direction of the ECM device is located, it makes sense to form a notch at the transmit beampattern in the direction. Current transmit beampattern notching techniques for colocated Multiple-Input Multiple-Output (MIMO) radar mainly use waveform optimization methods, which however has a huge computation cost. In this paper, we propose an online MIMO transmit beampattern notching method that is formulated as a linear programming problem and then can support online implementation for its low computation cost. The basic concept is to slightly vibrate the phases of a group of MIMO waveforms optimized offline for certain purpose such that transmit beampattern notching would not spoil too much properties of original waveforms. Numerical results indicate that for an omnidirectional MIMO transmit beampattern formed with 4 transmit waveforms each with 128 codes, this method can form a -22dB transmit beampattern notch, whereas the peak sidelobe level grows merely 1.25dB.**

I. INTRODUCTION

MUltiple-Input Multiple-Output (MIMO) radar [1] research receives intensive interest in recent years. In contrast to its phased-array radar counterpart, colocated MIMO radar would transmit different waveforms through its elements, which provides more degrees of freedom to enhance performance with the same number of elements [2]. Waveform optimization is a hot topic in colocated MIMO radar research [3], [4] and thanks to more degrees of freedom, more flexible transmit beampatterns are achieved, which greatly enriches radar operation modes in real applications.

Current MIMO waveform optimization algorithms are generally implemented offline, because most of them need huge computation cost, especially those with range sidelobe suppression. Some algorithms have been proposed to save the computation cost [5], but online implementation is still a challenge. In real applications, it makes special sense in some situations to be adaptive with radar operation circumstances. Consider a situation where a radar operates against a hostile Electronic Counter Measurement (ECM) device [6]. Once radar signals are detected by the ECM device, the ECM device can transmit many deceptive signals or high power jamming signals to prevent the radar device from working. As the direction of ECM is easy to measure, an efficient method to combat with the threat is to form a notching at the transmit beampattern in the direction of ECM, so that less transmit power may not be detected. As an ECM may be present in any direction, it is favorable that the notch can be formed online instead of offline. In this case, we have scare waveform optimization algorithms at hand.

For phased-array radar, an online transmit beampattern notching algorithm [7] is presented that slightly adjusts initial phases of transmit antennas such that a notch can be formed at certain direction. Moreover, this optimization problem is formulated to be a linear programming algorithm to save computation cost and support online implementation. In this paper, we extend the works to colocated MIMO radar waveforms and study how to form a notch with existing MIMO waveforms.

For MIMO radar waveform optimization, there are mainly two strategies, both capability of forming notches at the transmit beampattern. One first optimizes a waveform covariance matrix [3], [8] and then optimizes waveforms matching the resulting waveform covariance matrix; the other directly optimize transmit waveforms matching a desirable transmit beampattern [4]. By controlling the desirable transmit beampattern, one can control the location where a notch is present. For both method, the problem is its huge computation cost. Once range sidelobes are involved, the optimization problem becomes nonconvex and the computation cost would be too huge to support online implementation. Consequently, we consider how to form a notch at the transmit beampattern of a group of MIMO waveforms designed offline.

It also presents the transmit beampattern optimization model. And its solution for covariance matrix is gradient algorithm. And a classic pattern matching design model is proposed in [3], meanwhile minimizing sidelobe beampattern design model is also proposed. The methods above are all achieve transmit beampattern notching by directly optimizing covariance matrix. And they all need huge computation cost. In order to improve this problem, MIMO radar transmit beampattern design without synthesising the covariance matrix is proposed [9], [10]. It achieves transmit baempattern design through two steps. In the first step, a waveform covariance matrix is synthesized. In the second step, to realize this covariance matrix actual waveforms are designed. Then the paper converts the two-step constrained optimization problem into a one-step unconstrained optimization problem. And for the optimization problem of MIMO radar transmit beampattern under jamming, a new MIMO radar waveform design method based on quadratically spatial and spectral optimization is proposed [11]. And the computational burden of the spatial optimization of waveforms can be reduced. In order to save more computation cost, an efficient colocated MIMO radar waveform design method based on two-step optimizations in the spatial and spectral is proposed [12]. And it can obtain the desired spectral notching without influencing the shape of the optimized beampattern.

The above methods all design transmit beampattern by optimizing the covariance matrix directly or indirectly. And they all have a huge computation cost. When the direction of the ECM device is located, it is easy to obtain a notch. But once the direction of jamming is unknown, it is difficult to have a notch on line. And it is not conductive to suppressing jamming. In order to solve the problems above, we propose an online MIMO transmit beampattern notching method that is formulated as a linear programming problem and then can support online implementation for its low computation cost. And we perturb the phase of transmit signal to avoid optimizing the covariance matrix. Meanwhile the model is proposed under minimizing the phase perturbation. And it reduces the computation cost.

II. Signal Model

Consider a colocated MIMO radar system with N transmit antennas. And the system simultaneously transmits N waveforms, denoted by $\mathbf{s}_n \in \mathbb{C}^{L\times 1}$ with L being the symbol length or number of subpulses, where $n = 1, 2, \cdots, N$ and $\mathbf{s}_n = [\mathbf{s}_n(1), \cdots, \mathbf{s}_n(L)]^{\mathrm{T}}$. So the transmit waveforms of the colocated MIMO radar can be denoted by $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_N]$. And the covariance matrix of the transmit waveforms can be written as

$$\mathbf{R} = E\left\{\mathbf{s}^*(l)\mathbf{s}^{\mathrm{T}}(l)\right\} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{s}^*(l)s^{\mathrm{T}}(l) = \frac{1}{L}\mathbf{S}^{\mathrm{H}}\mathbf{S} \quad (1)$$

where $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{\mathrm{T}}$ denote the conjugate transpose and the transpose. Assuming that the transmit waveforms of all array elements are narrow-band signals, the received signal at the far field θ is

$$\mathbf{r}(\theta) = \mathbf{a}^{\mathrm{H}}(\theta)\mathbf{s}(l) \quad (2)$$

where $\mathbf{a}(\theta) = [1, e^{j2\pi d\sin\theta/\lambda}, \cdots, e^{j2\pi d(N-1)\sin\theta/\lambda}]^{\mathrm{T}}$ is the corresponding steering vector for the transmitting antenna array, d represents the space between the transmit elements, and λ denotes the wave length. The average power of the signal $r(\theta)$ in the L subpulses is

$$\boldsymbol{P(\theta)} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{s}(l)\mathbf{s}^{\mathrm{H}}(l)\mathbf{a}(\theta) = \mathbf{a}^{\mathrm{H}}(\theta)\mathbf{R}\mathbf{a}(\theta) \quad (3)$$

If there is interferences in the θ_k direction, where $k = 1, 2, \cdots, K$ represents the number of interferences. The phase of the transmit waveforms $\mathbf{S}$ is disturbed. And the perturbation matrix is used to represent $\mathbf{\Phi}$, and $\mathbf{\Phi} = [\varphi_1, \varphi_2, \cdots, \varphi_N] \in \mathbb{C}^{L\times N}$, where $\varphi_n = [\varphi_{n1}, \varphi_{n2}, \cdots, \varphi_{nL}]^{\mathrm{T}}$ and $n = 1, 2, \cdots, N$. After the phase perturbation, the transmit waveforms is denoted by $\tilde{\mathbf{S}}$, and $\tilde{\mathbf{S}} = \mathbf{S}\odot\exp(j\mathbf{\Phi})$, where $\odot$ represents Hadamard product. Therefore, the response of the transmit signal in the direction of interference is

$$\boldsymbol{P(\theta_k)} = \mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{R}}\mathbf{a}(\theta_k) \quad (4)$$

where $\tilde{\mathbf{R}}$ represents the covariance matrix of the transmit waveform matrix $\tilde{\mathbf{S}}$ after the phase perturbations.

In order to suppress the interference in the θ_k direction, it is necessary to make the radar transmit beampattern form a notch in the direction. And the following constraint needs to be satisfied

$$\boldsymbol{P(\theta_k)} = \mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{R}}\mathbf{a}(\theta_k) = 0 \quad (5)$$

According to the minimum phase perturbation criterion, the following optimization model is constructed to make the interference direction form a notch

$$\begin{aligned} \min_{\varphi_{nl}} \quad & \sum_{l=1}^{L}|\varphi_{nl}| \\ s.t. \quad & P(\theta_k) = \mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{R}}\mathbf{a}(\theta_k) = 0 \end{aligned} \quad (6)$$

where $|.|$ denotes modulus.

The model can effectively suppress the interference with a notch in the direction of the interference. And the optimization model is a nonlinear programming problem with a huge computation cost. So it can not form a notch online. When the location of interference changes, the model has no good effect on suppressing interference. In order to improve these problems, we convert nonlinear model to linear model and solve it.

A. Solution

Consider simplifying the constraint of formula (5) firstly. And for the transmit waveforms $\tilde{\mathbf{S}}$ after perturbation, the constraint of formula (5) can be expressed as

$$\mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{S}}^{\mathrm{H}}\tilde{\mathbf{S}}\mathbf{a}(\theta_k) = 0 \quad (7)$$

Let $\tilde{\mathbf{s}}(l)$ represents the transmitting signal vector at the moment l. That is, it is the lth column of the perturbed transmit waveforms $\tilde{\mathbf{S}}^{\mathrm{T}}$, where $\tilde{\mathbf{s}}(l) = [\tilde{s}_1(l), \tilde{s}_2(l), \cdots, \tilde{s}_N(l)]^{\mathrm{T}}$. So the constraint of formula (7) can be rewritten as

$$\mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{s}}(l) = 0 \quad (8)$$

The waveform obtained by the constraint (8) can form sharp notches in the interference direction. And it can be seen as N transmit antennas transmitting signals at the same time. So the constraint of formula (8) is equivalent to considering the MIMO radar as a combination of N arrays. Therefore, as long as each transmit antenna can form a notch in the interference direction, the transmit beampattern of MIMO radar can suppress the interference.

Based on the above derivation, the simplified optimization model is obtained

$$\begin{aligned} \min_{\varphi_{nl}} \quad & \sum_{l=1}^{L} |\varphi_{nl}| \\ s.t. \quad & \mathbf{a}^{\mathrm{H}}(\theta_k)\tilde{\mathbf{s}}(l) = 0, k = 1, 2, \cdots, K \end{aligned} \tag{9}$$

The simplified optimization model is still a nonlinear programming problem. And it needs to be transformed to the next step. Taking into account the relatively small number of interferences, namely $K \ll N$, the phase perturbations expected are also relatively small, and it would require $|\varphi_{nl}| \ll 1$. In this case, constraint function in (9) can be expanded in Taylor's first order [7]. And the optimization model is rewritten as

$$\begin{aligned} \min_{\varphi_{nl}} \quad & \sum_{l=1}^{L} |\varphi_{nl}| \\ s.t. \quad & \mathbf{a}^{\mathrm{H}}(\theta_k)\mathbf{s}(l) + j\sum_{l=1}^{L} [\mathbf{a}^{\mathrm{H}}(\theta_k)\mathbf{s}(l)]\varphi_{nl} = 0 \end{aligned} \tag{10}$$

At this point, the problem is converted into a linear programming problem with an analytic solution [13], [14]. Compared with the equation (6), the computational complexity is reduced. And the transmit beampattern obtained from the above formula is the transmit beampattern after suppression the interferences. Also, when interferences exist, we can obtain a notch quickly. And thus the iterferences will be suppressed in time. But all of the above assumptions are based on the ideal, that is, there is no noise in the transmit signal. So further study is needed.

III. Numerical results

In order to demonstrate the method proposed in this paper, the method is simulated. Assuming the colocated MIMO radar have four transmit antennas with a symbol length of 128 and an interference direction of 25 degrees. The cancellation beampattern are shown in Fig.1. And it can be seen that there is a null of -22.03dB in the direction of interference, which shows the theoretical validity of this method. Fig.2 shows a comparison of the auto correlation of angular waveform before and after phase perturbations. We can see the peak sidelobe level grows merely 1.25dB, which indicates a small change in the peak sidelobe before and after the phase disturbance. Although the above simulation results show the effectiveness of the proposed method, the antennas above are all omni-direction antenna. So in order to illustrate the effect of the method better, we consider a directional antenna. Suppose the directions of the transmit antennas are -40 degrees and 40 degrees. And the MIMO radar have 10 transmit antennas with a symbol length of 128 and an interference direction of 5 degrees. The cancellation beampattern is shown in Fig.3. And it can be seen that there is a null of -109.2dB in the direction of interference, it further proves the effectiveness of this method. And Fig.4 shows a comparison of the the auto correlation of angular waveform before and after phase perturbations. The phase perturbations also has little influence on the peak sidelobe with 1.54dB rising. The above two simulations fully demonstrate the validity of the method proposed in notching, and apply to omnidirectional antennas and directional antennas.

The proposed method of beampattern notching directly adjusts the phase of the transmitted signal, thereby avoiding the optimization of the covariance matrix of the transmitted signal. And thus the method proposed has a low computation cost. The proposed method uses Taylor's first-order expansion to convert nonlinear constraints into linear constraints, which further reduces the computation cost. We solve the problem through direct solution of standard linear programming (SLP). For the problem in this paper, the MIMO radar contains N transmit array elements, the length of the transmitted signal symbol is L, and there are k interferences in the scene. It can be known from the optimization equation that there are N equations and L variables. So according to the SLP direct solution method. The principle can be calculated as the computation cost of $\mathbf{O}(NL^2)$ or $\mathbf{O}(L(L-N)^2)$.

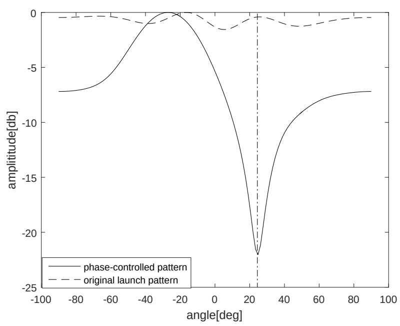

Fig. 1. transmit beampattern before and after phase disturbed

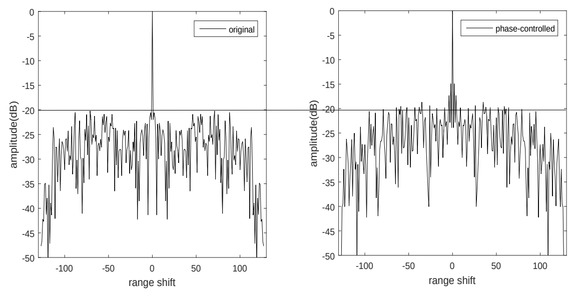

Fig. 2. auto correlation of angular waveform before and after phase disturbed

IV. Conclusion

In this paper, a collocated MIMO radar on-line emitter method based on small phase perturbations can be used to

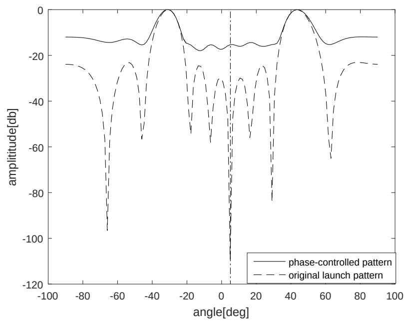

Fig. 3. transmit beampattern before and after phase disturbed

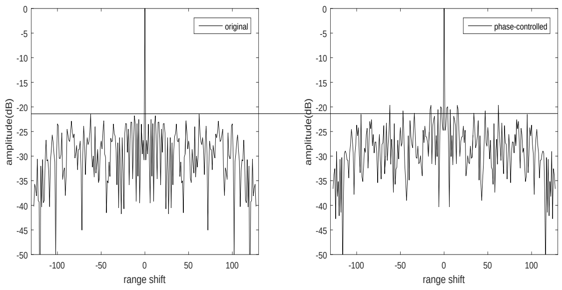

Fig. 4. auto correlation of angular waveform before and after phase disturbed

suppress the unknown interference sources in practical applications. During the design process, the phase of the transmitting signal is disturbed, and the response of the disturbance direction is zero under the minimum principle of the disturbance, so as to achieve the purpose of suppressing the interference. The design criterion is a linear programming problem. The simulation results show the effectiveness of the method of interference suppression, and ensure the sidelobe. Compared with the existing adaptive zero method, this method avoids the optimization of signal correlation matrix and reduces the amount of calculation, so it can quickly produce zero online. However, this paper does not consider the coupling effect of the emission matrix, which needs further study.

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