Space-Range-Doppler Focus Processing: A Novel Solution for Moving Target Integration and Estimation Using FDA-MIMO Radar

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***Abstract*—Frequency diverse array (FDA) is an emerging array technique that employs a small frequency increment across its array elements to produce a range-angle-dependent beampattern, which provides promising applications for joint angle-range estimation of targets. However, the FDA has several problems for signal processing, such as angle and range coupling and Doppler integration, and few papers deal with FDA for moving target. In this paper, we combine the FDA and MIMO technique and propose the concept of space (angle)-range-Doppler (SRD) focus processing, which is a novel solution for moving target integration and estimation. It utilizes the property of FDA and high-resolution Doppler processing of MIMO. Based on the data model of FDA-MIMO radar, we provide a practical method for SRD processing via A&D joint estimation and sparse time-frequency distribution (STFD). Both theoretical and numerical simulation results verify that proposed method has better ability for joint range-angle-Doppler processing, which can be used for low-observable moving target detection and estimation under complex environment**

Keywords—Frequency diverse array (FDA) radar; Space-Range-Doppler (SRD) focus processing; Moving target integration; Sparse time-frequency distribution (STFD)

I. INTRODUCTION

Radar, as one of the primary means of target detection and recognition, is widely used in public and defense security fields. However, due to the complex background environment and the complex movement characteristics of the target, the target's radar echo is extremely weak and complex, making the radar's detection performance of moving targets difficult to meet the actual needs [1][2]. Low observable moving target detection technology in complex background has become the key constraint factor affecting the performance of radar and is also a worldwide difficulty. The key point is that in both time and the frequency domain, the target's energy is very weak resulting in a low signal-to-noise/clutter ratio (SNR/SCR). One possible and practical way is to accumulate target's energy from different domains [3]. However, due to the working system, traditional radar can only do this in range and Doppler domain separately. Therefore, it is necessary to investigate new radar system and radar technology for moving target integration in order to meet the challenge of complex environment and target.

In the past few years, multiple-input multiple-output (MIMO) radar theories have been well developed [4]. The flexibility of MIMO radar in signal waveform and array configuration can provide more freedom degrees for signal processing in complex and dynamic clutter background. At the same time, it can extend the target dwell time and improve Doppler resolution, which is helpful for energy accumulation and clutter suppression. However, the MIMO radar also has its inherent drawback: the beam direction is not related to the distance. However, in some applications, such as interference or clutter suppression, it is often expected that the array beams can point to different distances at the same angle within the same snapshot, which requires that the beam direction can be varied with distance.

Recently, a flexible antenna array named frequency diverse array (FDA) is receiving an increasing amount of attention [5]. The most important difference from conventional phased arrays is that a small frequency increment, compared to the carrier frequency, is applied across the elements. The frequency increment results in that the beam direction changes as a function of the range, angle, and time. It can be considered as a specific transmit beamforming applied on the phased-array. It provides many promising applications [6], such as joint angle and range localization, radio frequency (RF) stealth, anti-clutter, anti-jamming, and so on. However, transmit beampattern of FDA is coupled in the range-angle domains, which is a difficulty for signal integration and estimation. The most practical decoupling approach is to jointly utilize FDA and MIMO technique. In doing so, both range and angles of targets can be unambiguously estimated due to the increased degrees-of-freedom (DOFs) provided by the MIMO technique. The FDA radar can obtain controllable DOFs in the transmit-receive dimensions, thus leading to range-angle-dependent beampattern in the receiver. This provides possible applications which are difficult for the traditional phased-array radar. In 2017, "IEEE Journal of Selected Topics In Signal Processing" published a special issue of "Time/Frequency Modulation Array Signal Processing" and focused on FDA−MIMO beamforming, parameter estimation, etc. [5][7].

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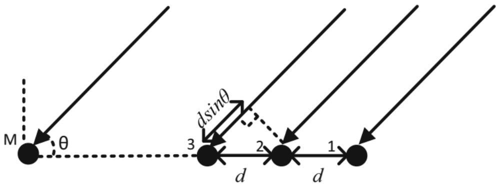


Fig.1. A uniform linear FDA with M elements.

In 2017, the IEEE International Radar set up the special session of FDA, which attracted great attention of experts and scholars. At present, the research on target detection of frequency array radar is mostly aimed at stationary targets. Different from conventional target integration methods, where the target range, angle and Doppler are measured separately, FDA radar has the potential ability of joint angle-range-Doppler estimation with slow-time sampled pulses in each coherent processing interval (CPI) [8][9]. However, for the moving targets, the time-varying characteristics of Doppler makes the energy divergence. How to make full use of the space, time and frequency resources of FDA-MIMO radar to improve the detection and estimation performances for moving target needs further investigation.

In this paper, we propose a novel solution for moving target integration and estimation using FDA-MIMO radar, which is named as space (angle)-range-Doppler (SRD) focus processing. The data model of FDA radar is established in section □. In section □, we introduce the principle of SRD focus processing of FDA-MIMO radar and give an example for its implementation using sparse time-frequency distribution (STFD). Moreover, the detection and estimation procedure for moving target is introduced. Finally, simulation results and conclusions are provided in sections IV and V, respectively.

II. DATA MODEL OF FDA RADAR

Consider a uniform linear array (ULA) with M transmit elements and N-antennas receiving ULA and whose inter-element spacing is d, as shown in Figure 1. Take the first transmit antennas as the reference point and the carrier frequency at the m th antenna is [5]

$$f_m = f_0 + \Delta f_m \tag{1}$$

where f_0 is the radar carrier frequency, $\Delta f_m = (m-1)\Delta f$ is the frequency increment which can be ignored compared with the carrier frequency.

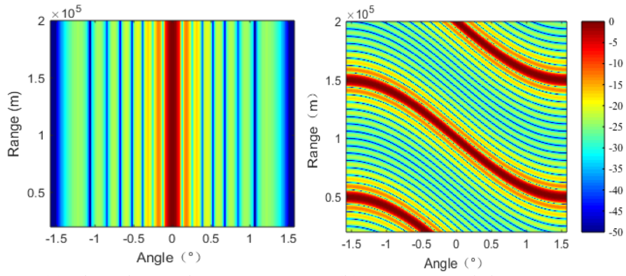
The transmitted signal of the mth antenna can be written as

$$s_m(t) = w_m \mathrm{e}^{\mathrm{j}2\pi f_m t}, m = 0, \cdots, M-1 \tag{2}$$

where w_m is the transmit weight. The synthesized signals arriving at a given far-field point target with angle-range pair (r, θ) can then be expressed as

$$S(t;r,\theta) = \sum_{n=0}^{M-1} w_m^* s_m(t-\tau_m) \tag{3}$$

where $\tau_m = r_m / c_0$ denotes the signal propagation delay from the mth element to the target, with c_0 and $r_m = r - md\sin\theta$ being the speed of light and target slant range, respectively.



(a) Phased-array beampattern (b) FDA transmit beampattern

Fig.2. Transmit beampattern comparisons between FDA and phased-array.

Following the narrow-band assumption, then (3) can be approximately expanded as

$$\begin{aligned} S(t;r,\theta) &= \sum_{n=0}^{M-1} w_m^* \mathrm{e}^{-\mathrm{j}2\pi(f_0+\Delta f_m)\left(t-\frac{r-md\sin\theta}{c_0}\right)} \\ &\approx \mathrm{e}^{-\mathrm{j}2\pi f_0(t-r/c_0)} \sum_{n=0}^{M-1} w_m^* \mathrm{e}^{-\mathrm{j}2\pi\Delta f_m(t-r/c_0)} \mathrm{e}^{-\mathrm{j}2\pi f_0 md\sin\theta/c_0} \end{aligned} \tag{4}$$

The array factor at the position (r‹ □) can be expressed as

$$\mathrm{AF}(t;r,\theta) = \sum_{n=0}^{M-1} w_m^* \mathrm{e}^{-\mathrm{j}2\pi\Delta f_m(t-r/c_0)} \mathrm{e}^{-\mathrm{j}2\pi f_0 md\sin\theta/c_0} \tag{5}$$

Supposing that M=16, Δf=3kHz, and f_0 =3.5GHz, Figure 2 compares the FDA transmit beampattern with a traditional phased-array. It is well recognized that FDA creates a range-angle-dependent beampattern whose amplitude and spatial distribution can be controlled by changing the frequency increments and the number of array elements[6]. Since FDA has the coupling problem in the range and angle domain, FDA and MIMO technique are jointly exploited for range-angle estimation of target, where the range changing characteristics are produced by FDA and the angle changing characteristics are provided by MIMO.

III. PRINCIPLE OF SPACE-RANGE-DOPPLER (SRD) FOCUS PROCESSING FOR FDA-MIMO RADAR

Considering that the electromagnetic signals are independently propagating in free space, the far-field signal received by the nth element can then be represented by

$$y_{m,n}(r,\theta) = \alpha(r,\theta)\mathrm{e}^{-\mathrm{j}4\pi\Delta f_m r/c}\mathrm{e}^{\mathrm{j}2\pi md/\lambda\sin\theta}\mathrm{e}^{\mathrm{j}2\pi nd/\lambda\sin\theta} + n_{m,n} \tag{6}$$

where $\alpha(r,\theta)$ is the complex reflection coefficient, λ is the wavelength, $n_{m,n}$ is the additive noise.

After matched filtering, the returned signal reflected by the far-field target can be concisely expressed by vector form,

$$\mathbf{y} = \alpha(r,\theta)\left[\mathbf{a}_\mathrm{R}(r) \odot \mathbf{a}_\mathrm{t}(\theta)\right] \otimes \mathbf{a}_\mathrm{r}(\theta) + \mathbf{n} \tag{7}$$

where $\odot$ and $\otimes$ denote the Hadamard (element-wise) product and Kronecker product operators respectively. $\mathbf{a}_\mathrm{t}(\theta)$, $\mathbf{a}_\mathrm{r}(\theta)$, and $\mathbf{a}_\mathrm{R}(r)$ are the transmit, receive, and transmit range steering vectors, respectively. They have the forms of

$$\mathbf{a}_\mathrm{t}(\theta) = \left[1 \quad \mathrm{e}^{\mathrm{j}2\pi df_0/c\sin\theta} \quad \cdots \quad \mathrm{e}^{\mathrm{j}2\pi(M-1)df_0/c\sin\theta}\right]^T \tag{8}$$

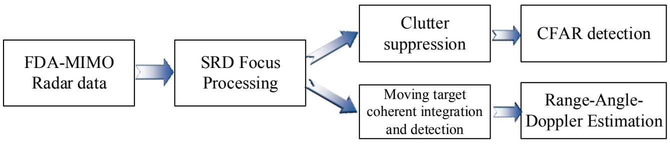


Fig.3. FDA-MIMO radar signal processing via SRD focus processing.

$$\mathbf{a}_{\mathrm{r}}(\theta)=\left[1 \quad \mathrm{e}^{\mathrm{j}2\pi d f_0/c\sin\theta} \quad \dots \quad \mathrm{e}^{\mathrm{j}2\pi(N-1)df_0/c\sin\theta}\right]^T \tag{9}$$

$$\mathbf{a}_{\mathrm{R}}(r)=\left[\mathrm{e}^{-\mathrm{j}4\pi\Delta f_0 r/c} \quad \mathrm{e}^{-\mathrm{j}4\pi\Delta f_1 r/c} \quad \dots \quad \mathrm{e}^{-\mathrm{j}4\pi\Delta f_{M-1} r/c}\right]^T \tag{10}$$

where, ^{T} is conjugate transpose operator. As shown in (7), the transmit steering vector is not only angle-dependent but also range-dependent, i.e.,

$$\mathbf{y} \triangleq \alpha(r,\theta)\mathbf{a}(r,\theta)+\mathbf{n} \tag{11}$$

Therefore, the FDA-MIMO radar could utilize the transmit and receive DOFs to determine the range and angle of the target. Now, we extend the above model for Doppler information, i.e., if there is a moving target. Suppose the hypothetical target has a radial velocity v_r, which results in a Doppler frequency shift f_d in the returned signals. Suppose the frequency increment Δf adopted in the FDA-MIMO radar is relatively negligible while compared with f_0 and the waveform bandwidth, and thus, the Doppler frequency shift due to Δf can be ignored. Following (7), the signal model for the moving target can be revised as

$$\begin{aligned}\mathbf{y}_{\mathrm{mov}} &= \alpha(r,\theta,f_{\mathrm{d}})\left[\mathbf{a}_{\mathrm{d}}(f_{\mathrm{d}})\otimes\mathbf{a}(r,\theta)\right]+\mathbf{n}\\ &=\xi\boldsymbol{\mu}_{\mathrm{mov}}(r,\theta,f_{\mathrm{d}})+\mathbf{n}\end{aligned} \tag{12}$$

where $\mathbf{a}_{\mathrm{d}}(f_{\mathrm{d}})$ is the Doppler vector in the slow time, given by

$$\mathbf{a}_{\mathrm{d}}(f_{\mathrm{d}})=\left[1 \quad \mathrm{e}^{\mathrm{j}2\pi f_{\mathrm{d}}T_p} \quad \dots \quad \mathrm{e}^{\mathrm{j}2\pi(L-1)f_{\mathrm{d}}T_p}\right]^T \tag{13}$$

$$\boldsymbol{\mu}_{\mathrm{mov}}(r,\theta,f_{\mathrm{d}})=\mathbf{a}_{\mathrm{d}}(f_{\mathrm{d}})\otimes\mathbf{a}(r,\theta) \tag{14}$$

where L is the sample number. And $f_{\mathrm{d}}=2v_0/\lambda$ if the target moves with constant velocity, and $f_{\mathrm{d}}=2(v_0+a_s t_m)/\lambda$ if the target moves with accelerated velocity a_s.

In fact, $\boldsymbol{\mu}_{\mathrm{mov}}(r, \theta, f_{\mathrm{d}})$ can be viewed as a joint angle-range Doppler steering vector in the slow time. It can be seen from the above equation that the joint estimation of SRD can be realized by using FDA-MIMO radar for moving targets. Combined with the long-time degree of MIMO, the Doppler resolution of the echo signal can be greatly improved. Therefore, FDA-MIMO radar has SRD three-dimensional focus processing capabilities, which helps further improve the SNR/SCR and obtain the refined characteristics of the moving target.

The processing flow architecture of SRD focus processing for FDA-MIMO radar is shown in Figure 3. Then, multi-dimensional joint coherent integration of the signal is completed, and the SCR gain can be improved. In other words, SRD focus processing can integrate multiple procedures of conventional radar signal processing, such as pulse compression, angle measurement, and Doppler filtering. It can achieve high-precision measurement of target parameters. The decoupling process of FDA-MIMO can be regarded as the focusing process in angle and range.

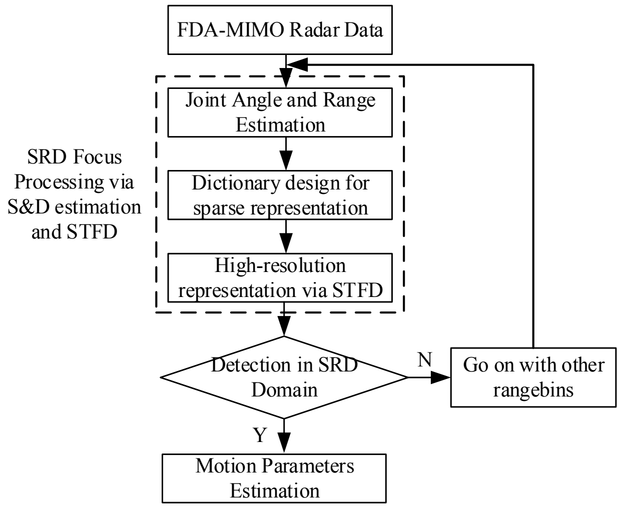


Fig.4. Flowchart of SRD focus processing for FDA-MIMO radar via A&D joint estimation and STFD.

Due to the complex motion of moving target, the Doppler may exhibit time-varying properties, and during long observation time, the computational cost of SRD focus processing may increase exponentially. Then how to obtain high integration gain and Doppler resolution of moving target and at the same time reduce the computational cost is a key point for SRD focus processing of FDA-MIMO radar. Recently, STFD has been proved as a promising tool to be competent for this task. The merits of TFD and sparse representation are combined together, and several STFD methods are proposed [10], such as sparse Fourier transform (SFT) , sparse fractional FT (SFRFT) [11], and sparse fractional ambiguity function (SFRAF) [12], etc., dealing with uniformly moving target, accelerated moving target, and maneuvering target with jerk motion. Figure 4 gives the flowchart of SRD focus processing for FDA-MIMO radar via S&D joint estimation and STFD. It mainly consists of four steps, i.e.,

a) Joint range and angle estimation. Due to the adoption of orthogonal waveforms, the coupling of the radiation pattern in the distance and azimuth is destroyed, so that the FDA-MIMO has a natural decoupling capability. Then, MUSIC and other algorithms can be employed to achieve target integration and positioning in range and angle dimensions.

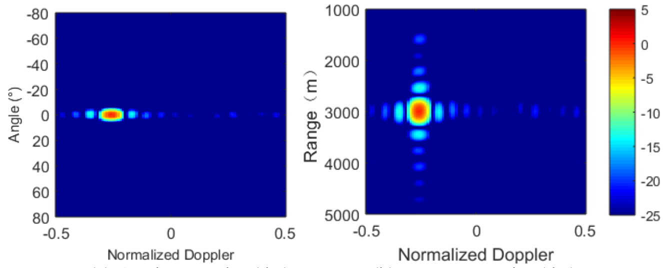
b) Perform STFD in Doppler domain, which consists of two steps, i.e., proper dictionary design, optimization and sparse representation.

c) Moving target detection in SRD domain, which can use constant false alarm rate (CFAR) method.

d) Motion parameters estimation, such as velocity, acceleration, jerk motion, et al..

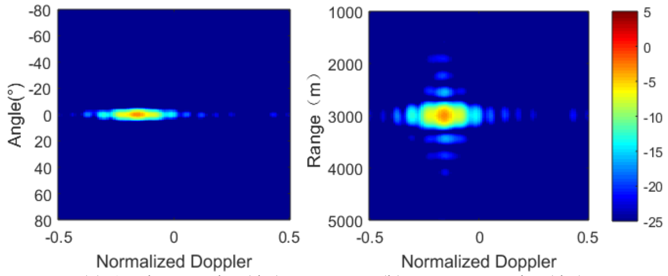
IV. Simulation Results

In this section, computer experiments are carried out to evaluate the effectiveness of the proposed SRD focus processing for FDA-MIMO radar. In the simulations, we consider that the FDA-MIMO radar transmits continuous signal, including M=16 transmit elements and N=16



(a) Angle-Doppler (dB) (b) Range-Doppler (dB)

Fig.5. FDA-MIMO spatial spectrum of uniformly moving target (SNR=0dB).



(a) Angle-Doppler (dB) (b) Range-Doppler (dB)

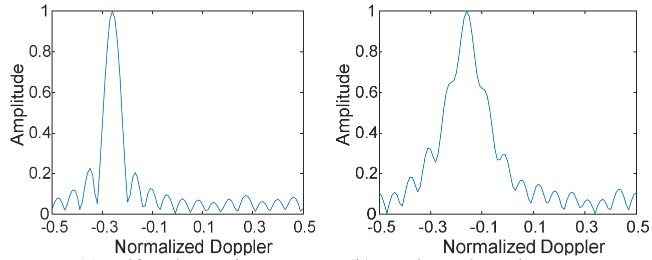
Fig.6. FDA-MIMO spatial spectrum of accelerated moving target (SNR=0dB).

receive elements. A total of L samples are collected during the coherent processing interval. The reference carrier frequency is f_0=1GHz while the frequency increment of FDA is Δf=30kHz. The SNR=0dB, and the target is assumed at the slant range 3km with two uniformly velocity and accelerated velocity, respectively.

Figure 5 and 6 show spatial spectrum of FDA-MIMO radar for moving target with uniformly and accelerated velocity, including angle-Doppler spectrum and range-Doppler spectrum, respectively. It can be seen that the Doppler spectrum of the FDA-MIMO radar is different from that of the stationary target, and the moving target has a Doppler characteristic corresponding to the motion status. The Doppler spectrum of the non-uniformly moving target diverges across multiple Doppler bins which can be found in figure 7. It is not conducive to the accumulation of target energy and the estimation of motion parameters. Then we perform the SRD focus processing according to the flowchart of figure 4, and SFT and SFRFT are employed for Doppler integration. From figure 8, the sparsity of the Doppler signal component is remained. Hence, not only the energy is accumulated, but also the high-resolution of time-varying Doppler signal is obtained. From the above analysis, the FDA-MIMO radar outperforms conventional phased-arrays and MIMO radar, achieving energy integration and estimation of moving target in angle, range, and Doppler domains.

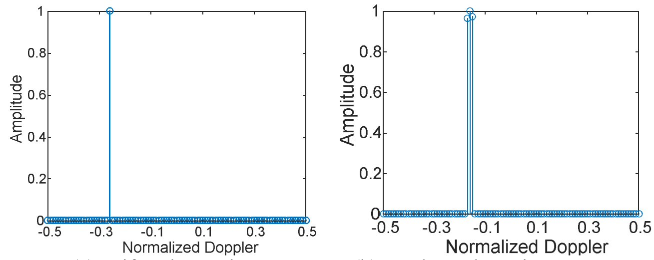
V. Conclusions

FDA-MIMO radar has received much attention in recent years due to its range-dependent transmit beampattern, but it has several problems for signal processing, such as angle and range coupling and Doppler integration. The main contribution of this paper is that we established the concept of SRD focus processing, which is a novel solution for moving target integration and estimation using FDA-MIMO radar. Moreover, we propose a practical method for SRD processing via STFD. Both theoretical and numerical results verify that FDA-MIMO indeed outperforms traditional and has better ability for joint range-angle-Doppler processing. In future work, we plan to



(a) uniformly moving target (b) accelerated moving target

Fig.7. FDA-MIMO spatial spectrum range-Doppler slice.



(a) uniformly moving target (b) accelerated moving target

Fig.8. FDA-MIMO spatial spectrum range-Doppler slice via SRD focus processing.

study the use of SRD focus processing for FDA-MIMO radar with other algorithms and do experiments with real data.

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