Sparse Fractional Fourier Transform and Its Applications in Radar Moving Target Detection

Xiaohan Yu*, Xiaolong Chen†, Yong Huang, Jian Guan
Naval Aviation University, NAU
Yantai, Shandong P.R. China, 264001
e-mail: cxlcxl1209@163.com†; yxhyxh0803@163.com*

***Abstract*—Attention has been focused on the radar moving target detection under limited radar resources conditions and complex environment. First, some key technology of sparse Fourier transform (SFT) is introduced in this paper. Then the implementation of sparse fractional Fourier transform (SFRFT) is briefly described. Most importantly, the SFRFT algorithm is applied to radar target detection, and a novel fast and effective method is proposed to improve the detection and parameter estimation performance of maneuvering target. The effectiveness of the method is verified by simulation experiments.**

Keywords—Radar signal processing; Sparse Fourier transform (SFT); Sparse fractional Fourier transform (SFRFT); Moving target detection; Motion parameter estimation

I. INTRODUCTION

Radar, which is one of the most important detection means for monitoring comprehension, the improvement of its moving target detection performance is of great importance [1]. However, due to the influence of complex background and complex movement characteristics of targets, the detection of moving targets, especially maneuvering targets, face a severe test. The signal-to-noise/clutter ratio (SNR/SCR) of maneuvering target is low, and the Doppler frequency of echo shows time-varying and non-stationary characteristics [2]. At present, the target detection methods can be roughly categorized into statistics-based detectors [3], nonlinear analysis approaches [4], time-frequency analysis methods, etc. Traditional statistics-based methods are based on the clutter statistical distribution model, and they cannot obtain expected results when the model is mismatched in the real complex environment. Characteristic parameters are adopted in the nonlinear analysis methods to distinguish clutter and targets, but the results may not be favorable in case of low SNR, and corrective detection of moving targets is difficult to be realized.

The time-frequency analysis methods provide conjoint distribution information in both time and frequency domain, which can describe the relationship of signal between time and frequency, so these methods are introduced to improve the detection performance of non-stationary signals. In general, echo signal of maneuvering target can be described as linear frequency modulation (LFM) signal [5]. The fractional Fourier transform (FRFT), which can be regarded as the rotation of the time-frequency plane, has a good energy concentration property of LFM signal at the certain rotation angel, and it has been widely used in moving target detection and parameter estimation [6], [7]. However, the parameter search-based FRFT method faced heavy complexity burden. And due to the large amount of echo data caused by wide observation range of radar and new system radar, such as multiple-input and multiple-output (MIMO) radar, it is urgent to study the effective and efficiency radar signal processing methods with high time-frequency resolution and suitable for large data volumes [8].

In recent years, a novel sub-linear fast algorithm for sparse signals snamed sparse Fourier transform (SFT) was developed by scholars at Massachusetts Institute of Technology (MIT) [9], [10], which is more efficient than the traditional fast Fourier transform (FFT). For an N-point input signal which has a large size with a sparse spectrum, SFT can reduce the complexity of FFT to $O(K\log_2 N)$, where K stands for the sparsity of signal [11]. Among the various kinds of discrete FRFT (DFRFT) algorithms, the least complex method is achieved by Pei's discrete method [12] and it will be further improved by exploiting the merits of the SFT scheme. Liu redesigned Pei's method on the basis of SFT and studied a new fast algorithm, namely sparse FRFT (SFRFT) [13], so the complexity of DFRFT is further reduced. The radar echo of target can be regarded as a superposition of a few strong scattering center echoes, which has a property of sparsity [14]. In reference [8], the theoretical framework of the sparse time-frequency distribution-based (STFD) target characteristic analysis method was established by introducing the local optimization idea of sparse decomposition into time-frequency analysis. On the basis of [8], the SFRFT algorithm is applied to radar signal processing in this paper to obtain the sparse fractional domain high resolution representation of target echoes. The computational efficiency, time-frequency resolution and parameter estimation accuracy can be effectively improved by implementing the target detection in sparse fractional domain. Thus, the radar detection performance under limited radar resources conditions and complex environment will be improved.

The rest of the paper is arranged as follows. In section Ⅱ, we introduce the implementation method of SFRFT. The detection process of the SFRFT-based method is given in Section Ⅲ. In Section Ⅳ, the effectiveness of the detection method is validated by simulation results. The last section concludes the paper and presents its future research direction.

II. Principle and Implementation of SFRFT

A. SFT

Since the SFRFT algorithm is proposed on the basis of SFT, the concept of SFT will be illustrated at first before we introduce the implementation of SFRFT. SFT is a class of sub-linear time algorithms for computing the DFT of a time domain signal which is sparse in the frequency domain, i.e. there are very few "large" coefficients in the frequency domain. The key idea of this algorithm is to bin the Fourier coefficients into a small number of buckets (the number is B). Since the signal is sparse in the frequency domain, each bucket is likely to have only one large coefficient [9]. Thus the N-point long sequence is converted into B-point short sequence and DFT is done on the latter. According to the calculation results, the buckets which do not contain large coefficients are negligible. Finally, reconstruction algorithm is designed according to the corresponding binning rules to restore the N-point original signal. The main techniques of SFT theory are as follows:

1) Permutation of Spectra: To make separation of nearby coefficients in frequency domain, a permutation is conducted by resetting the time domain signal $x(n)$, i.e.,

$$r(n) = x\left((\sigma \cdot n) \bmod N\right), n \in [1, N], \tag{1}$$

where σ is a random odd number that is invertible mod N and $\sigma \in [1, N]$. Suppose $X(m)$ and $R(m)$ are the frequency domain representations of $x(n)$ and $r(n)$ respectively, it can be proved that the relation of them is

$$R(m) = X\left((\sigma^{-1} \cdot m) \bmod N\right), m \in [1, N]. \tag{2}$$

2) Window functions: The binning is realized by using a filter f that is concentrated both in time and frequency domains, i.e., f is zero except at a small number of time coordinates, and its Fourier transform F is negligible except at a small fraction of the pass region. Here we choose the flat window function as filter f,

$$F(m) \in \begin{cases} \text{supp}(f) \subseteq [-\omega/2, \omega/2] \\ [1-\delta, 1+\delta], m \in [-\varepsilon' N, \varepsilon' N], \\ [0, \delta], \qquad m \notin [-\varepsilon N, \varepsilon N] \end{cases} \tag{3}$$

where ω is the length of window, ε' and ε denote the passband truncation factor and the stopband truncation factor respectively, δ denotes the extent of ripple oscillation.

3) Subsampled FFT: Define a signal $s(n) = r(n) \cdot f(n)$, $n \in [1, N]$, and suppose there is an integer B that can exactly divide N, i.e.,

$$\begin{aligned} Y(m) &= \text{FFT}\{y(n)\} \\ &= \text{FFT}\left\{ \sum_{i=0}^{\left\lfloor \frac{\omega}{B} \right\rfloor - 1} s(n + iB) \right\}, n \in [1, B] \end{aligned}, \tag{4}$$

$Y(m)$ is the frequency domain expression of $y(n)$, it can be proved that

$$Y(m) = S(m \cdot N/B), m \in [1, B], \tag{5}$$

which indicates that subsampling in frequency domain can be realized by aliasing in time domain.

B. SFRFT

According to Pei's discrete method [12], the DFRFT can be decomposed as two times of multiplication with chirp signals and one time of FFT if $\alpha \neq Q\pi$, α denotes rotation angle and Q is an integer. So the basic idea of SFRFT is to replace the FFT stage of FRFT with SFT. Assume that $z(n)$ is the original input signal, the $x(n)$ above can be expressed as

$$x(n) = z(n) \cdot \mathrm{e}^{\frac{\mathrm{j}}{2}\cot\alpha n^2 \Delta t^2}, n \in [1, N], \tag{6}$$

where Δt is the sampling interval of time domain.

After spectrum permutation, window function filtering and subsampled FFT, the value of each large coefficient are determined by the result of B-point FFT, and then the corresponding reconstruction methods are designed according to the binning rules to obtain the coordinates of each large coefficient in the original spectrum, so as to estimate more accurate approximations. One effective method is hash mapping, define a hash function

$$h_\sigma(m) = \lfloor \sigma \cdot m \cdot B/N \rfloor, \tag{7}$$

$h_\sigma(m)$ represents the binning rules which is decided by spectrum permutation and window function. Then we can find the position of large frequency coefficients by location loops. Put the dK coordinates of the maximum magnitudes in $Y(m)$ into the set J (d is the), output the preimage

$$I_r = \{ m \in [1, N] \mid h_\sigma(m) \in J \}. \tag{8}$$

For each $m \in I_r$, calculate the corresponding frequency values,

$$\hat{X}(m) = \frac{Y(h_\sigma(m)) \mathrm{e}^{-\mathrm{j}\pi o_\sigma(m)\omega/N}}{F(o_\sigma(m))}, m \in I_r, \tag{9}$$

where $o_\sigma(m) = \sigma \cdot m - h_\sigma(m) \cdot N/B$ denotes the offset. The estimated output $\tilde{X}(m)$ can be obtained by selecting the median values of $\hat{X}(m)$ getting in the estimation loops. Finally, we can obtain the SFRFT output $\hat{Z}_p(m)$ by multiplying another chirp function to $\tilde{X}(m)$, and p is the order of SFRFT.

III. SFRFT-BASED MANEUVERING TARGET DETECTION

On the condition that radar transmits single frequency signal or LFM signal, the radar echo can be approximated as LFM signal [7]. Assuming that the radar is working on tracking condition and a target is moving towards the radar at sea. The acceleration and initial velocity of targets are denoted a and v_0, respectively. Within observation time T, the model of single moving target in sea clutter can be characterised as follows:

$$z(t) = Si(t) + c(t) = A(t)\exp[\mathrm{j}2\pi f_0 t + \mathrm{j}\pi\mu_0 t^2] + c(t),\ |t| \le T, \quad (10)$$

where $Si(t)$ denotes the target signal, $A(t)$ is the signal amplitude, $\mu_0 = 2a/\lambda$ is the chirp rate, λ denotes the wavelength of the radar, $f_0 = 2v_0/\lambda$ is the center frequency corresponding to the velocity of the targets, and sea clutter is denoted as $c(t)$.

As shown in Fig.1, the SFRFT-based maneuvering target detection and parameter estimation method mainly consists of four steps, i.e.,

- Perform demodulation and pulse compression of radar returns, which achieves high-resolution in range direction;
- SFRFT, which mainly consists of spectrum permutation, window function filtering, subsampled FFT, reconstruction and two times of multiplication with chirp signals, it should be pointed out that if $\sin\alpha < 0$, the FFT operation in formula (4) need to be substituted with inverse FFT (IFFT);
- Traverse all distance search units, perform target detection in the SFRFT domain by comparing the output of SFRFT with an adaptive detection threshold;
- Feature extraction and parameter estimation. As shown in formula (11) and (12), the modulation frequency and center frequency are estimated according to the coordinates of the remaining peaks after SFRFT domain detection. And then, the target acceleration and initial velocity can be estimated by the relationships Mentioned above.

$$\{p_0, m_0\} = \arg\max_{p,m} \left|\hat{Z}_p(m)\right|, \quad (11)$$

$$\begin{cases} \hat{\mu}_0 = -\cot(p_0\pi/2) \\ \hat{f}_0 = m_0 \\ \hat{A}(t) = \mathrm{Re}\left[z(t)\exp\left(-2\mathrm{j}\pi\hat{f}_0 t + \mathrm{j}\pi\hat{\mu}_0 t^2\right)\right] \end{cases}, \quad (12)$$

where $\hat{f}_0$, $\hat{\mu}_0$ and $\hat{A}(t)$ are estimations of the center frequency, chirp rate and fluctuant amplitudes, respectively. Re{} represents the real part of the complex signal.

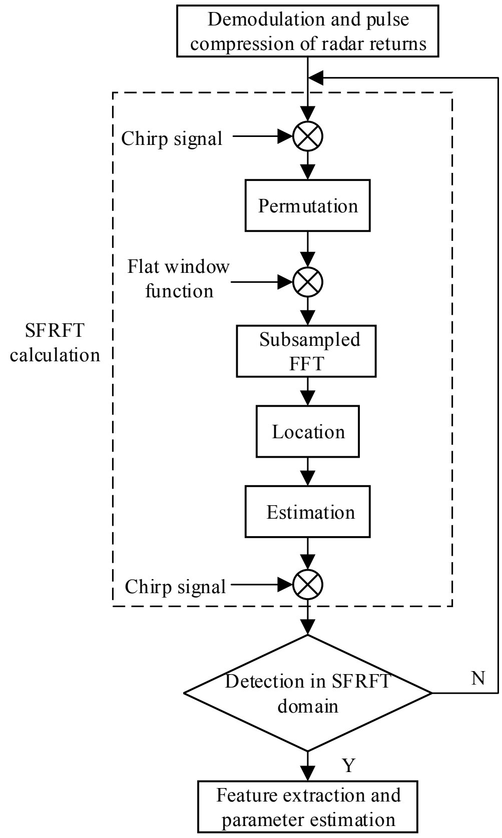


Fig. 1. Flowchart of SFRFT-based moving target detection method.

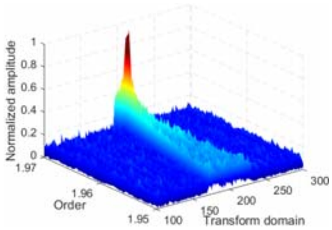
IV. EXPERIMENTAL RESULTS AND ANALYSIS

In this section, the detection performance of proposed method is verified by comparing the SFRFT domain maneuvering target detection method with Pei's sampling DFRFT detection method. Assuming that the radar system is a coherent system, its working wavelength is 3 cm. Under the background of complex Gaussian noise, the initial radial velocity of target is v_0=3 m/s and the maneuvering acceleration of target is a=0.3 m/s^2. The echo model is (10). The simulation satisfies the sampling theorem, the number of large coefficient is K=1, the sampling points and sampling frequency are N=8192 and f_s=1000 Hz, respectively. Table Ⅰ lists the specific simulation parameters, the calculation It can be obtained by calculation that the observation time T=8.192s, center frequency f_0=200 Hz, modulation frequency μ_0=20 Hz/s.

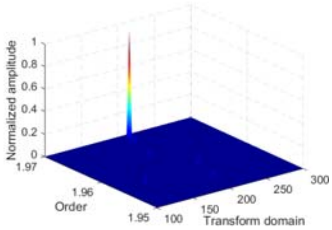
TABLE I. DESCRIPTION OF SIMULATION PARAMETERS

T(s)	f_s(Hz)	λ (m)	f_0(Hz)	μ_0(Hz/s)	K
8.192	1000	0.03	200	20	1

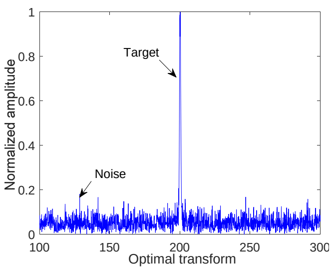
The SFRFT detector and DFRFT detector are respectively used to process the simulation signal. The step size of the transform order p is 0.0005 and the variation range is [1.8, 2.0]. The detection performance of the algorithm is analyzed in the Gaussian clutter background. Fig.2(a) and Fig.2(b) show the processing results of DFRFT and SFRFT, respectively. Fig.2(c) and Fig.2(d) show the optimal transformation results (p=1.968) of them (SNR=-3dB). It can be seen from the results that both DFRFT and SFRFT have good energy concentration for the target signal, but the SFRFT method can obtain higher time-frequency resolution and better clutter suppression performance.

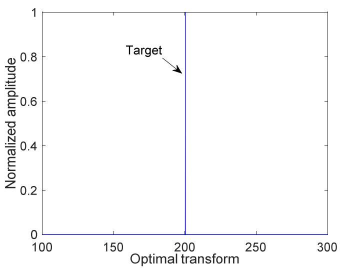


(a) Results of DFRFT



(b) Results of SFRFT





(c) The optimal transformation of DFRFT (p=1.968) (d) The optimal transformation of SFRFT (p=1.968)

Fig. 2. Comparison of DFRFT and SFRFT processing results (SNR=-3dB)

Furthermore, the detection performance of the algorithm is quantitatively analyzed. The absolute errors of estimated center frequency Δf_0 and estimated modulation frequency $\Delta\mu_0$ ($\Delta f_0 = \left|f_0 - \hat{f}_0\right|$, $\Delta\mu_0 = \left|\mu_0 - \hat{\mu}_0\right|$) are respectively calculated, and the calculation time is calculated. The comparison result (SNR=-3dB) is shown in Table Ⅱ. It can be seen that the SFRFT method achieves better clutter suppression performance at low SNR, and its parameter estimation accuracy is higher than the classical Pei sampling DFRFT method. In addition, DFRFT method is computationally intensive, SFRFT method can be adopted to improve its computational efficiency. In summary, the performance of maneuvering target detection can be effectively improved by the introduction of SFRFT. However, the robustness of the algorithm needs to be further enhanced due to many factors such as estimation method of large coefficients and parameters' setting of the filter window.

TABLE II. DETECTION PERFORMANCE COMPARISON OF SFRFT AND DFRFT (SNR=-3DB)

	Target peak	$\hat{f}_0$ (Hz)	$\hat{\mu}_0$ (Hz/s)	Δf_0 (Hz)	$\Delta\mu_0$ (Hz/s)	Time[a] (ms)
SFRFT	1	200.14	20.0226	0.14	0.0226	49.8
DFRFT	1	200.31	20.0226	0.3	0.0226	63.5

[a.] Computer configuration: Intel Core i7-4790 3.6GHz CPU; 16G RAM; MATLAB R2016a

V. CONCLUSIONS

Sparse time-frequency analysis method has the advantages of high efficiency, good time-frequency resolution, anti-clutter, which provides a new way for radar target detection. In this paper, the implementation process of sparse fractional Fourier transform (SFRFT) is described on the basis of sparse Fourier transform (SFT). The SFRFT is applied to radar moving target detection parameter estimation. Simulation experiment is carried out for validation. It is proved that the SFRFT-based detection method can achieve low computational complexity with good clutter suppression ability. And the performance of this detection method will be further improved by exploiting more superior window functions.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China (61501487, 61401495, U1633122, 61471382, 61531020), the Natural Science Foundation of Shandong (2015ZRA06052), the Aeronautical Science Foundation of China (20162084005, 20162084006, 20150184003), the Special Funds of Taishan Scholars of Shandong and Young Talents Program of CAST.

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