Target azimuth estimation in phased array radar using detected powers from main and side lobes

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***Abstract*—Target azimuth estimation in phased array radar with fixed antenna panel uses normally algorithms that act upon detected samples restricted to those from the main lobe of the antenna. The algorithms are mostly based on centroid methods that performs weighted average to calculate the center of intensity as the target azimuth. This paper presents a new algorithm to estimate target azimuth from all detected signals from main and side lobes acquired during antenna scanning. The algorithm estimates the target azimuth by finding the estimator of least variance. The estimators are defined to cover the range of all possible azimuth output, and are defined via a mathematical operation that includes the detected powers and reference prior-calculated powers. It is proven analytically that (always) the estimator that corresponds to the azimuth closest to the target azimuth contains the least variance. The simulations says the algorithm is more precise than the centroid algorithm.**

I. Introduction

Phased array radars determine the targets azimuths by pointing the antenna radiation pattern to different prior-defined directions, acquiring powers of detections from each pointing direction. In this scan process, each dwell time occurs with the antenna beam pattern fixed in the same direction, while the pulse is sent multiples times accordingly to a PRF. This paper draws its conclusion based in a radar operating in this configuration.

The fixed and finite number of directions of the phased array antenna beam patterns makes the azimuth estimation different from the radars with rotating antennas. While the radars with rotating antenna can calculate target azimuth with algorithms as binary integration [1] and ML algorithm [2], the radars with phased array antenna can not use these same algorithms.

One algorithm used for the phased array case is the centroid algorithm based on center of mass. It estimates the target azimuth by means of a weighted average of the detected powers from two or more adjacent detections; these detections must come from main lobes. The restriction of using detections from the main lobes comes from the fact that the azimuth estimation is based in an interpolation over the curve of one main lobe. The papers of [3] and in [4] showed that the performance of centroid approach can be improved if the antenna beam shape of the main lobes are modeled in a practical analytical mathematical form.

This paper proposes a new algorithm to estimate target azimuth that uses the powers detected of a target coming from either main or side lobes. It is based in identifying the estimator that has the least variance upon a family of estimators. Each estimator uses all detected powers and calculate an associated variance. The estimator with the least variance is the one corresponding to the target azimuth. It will be proved that there is always a best estimator whose variance tends to zero. The new algorithm is compared to the centroid implementation from the paper [5].

This paper is organized as follows. Section II describes the proposed algorithm. Section III discusses the radar equation fundamentals that permits the algorithm to work properly, and the best estimator to be unbiased and consistent. The algorithm is then presented in Section IV. Section V compares the current algorithm's features with the centroid's. Section VI explains how simulations were set up, and presents the results comparing the two algorithms. And finally section VII presents the conclusions.

II. Description of the algorithm

The proposed algorithm estimates the target azimuth based in a minimum variance unbiased estimator procedure. It is called the least variance estimator algorithm.

The family of estimators are $f(\mathbf{q}_\theta, \theta)$, where each chosen azimuth θ produces a new θ estimator. The random variable $\mathbf{q}_\theta$ of an θ estimator is defined in Eq. 1. The range of θ is all possible azimuths a target can be at in front of the antenna.

$$\mathbf{q}_\theta = 10\log(P^d + N) - 10\log(P^d_{ref,\theta}) \tag{1}$$

where $P^d + N$ is the power+noise received from a detected target when phased antenna array is pointing in the direction d, and $P^d_{ref,\theta}$ is the reference power of the direction d of the θ estimator.

As will be seen in more detail in section III, $P^d_{ref,\theta}$ is computed from the gains of the antenna radiation pattern.

The samples of the random variable $\mathbf{q}_\theta$ are those obtained from multiple detections of the same target when antenna points in different directions.

The θ estimator with least $var(\mathbf{q}_\theta)$ is chosen to represent the target azimuth estimation.

The variance of $\mathbf{q}_\theta$ is minimum when θ chosen is closest to the real target azimuth.

See Fig. 1 for a visual explanation of an example of the minimum variance identified at 0^0 degree among all variance of θ estimators.

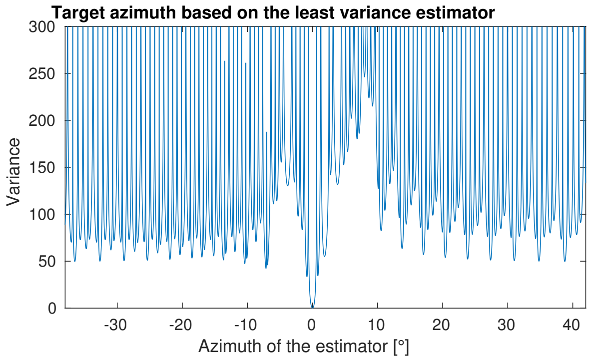

Fig. 1. Variances of the θ estimators when target is positioned at azimuth 0 degree.

The accuracy of the estimation is influenced by the choice of the family of estimators. Let δ_θ be the azimuth step: $\delta_\theta \in R$. The family of θ values can be defined as $\theta_0 + i\delta_\theta$ ($\theta_0 \in R$ and $i \in Z$); the azimuth output can only be one of these chosen values. The family of values can be chosen arbitrarily to fit the azimuth output needed.

The implemented algorithm when executing limits the number of estimators under evaluation, looking at only those estimators close to the azimuth of the maximum power received. This approach reduces computational cost.

III. Theoretical Fundamentals from the Radar Equation

Let Eq. 2 be the radar equation.

$$P^d_\theta = K \frac{P_t G^d_{t,az} G^d_{r,az} \sigma A}{(4\pi)^2 R^4} \tag{2}$$

where P^d_θ is the received target power when target is at azimuth θ relative to the antenna direction d, P_t is the radar transmission power, $G^d_{t,\theta}$ is the antenna transmission gain relative to the antenna direction d at the azimuth θ, $G^d_{r,\theta}$ is the antenna reception gain relative to the antenna direction d at the azimuth θ, σ is the target cross-section, A is the antenna aperture, R is the distance from the target to the radar, and K is a constant that represents other gains as signal processing gain, etc.

The values of $P^d_{ref,\theta}$ are those obtained from Eq. 2 when fixed values of σ and R are defined to an hypothetical target at azimuth θ as reference. Once such values are used during the least variance algorithm operation, the algorithm works as expected.

In the real case, the power received from a target contains noise as given in Eq. 3 and Eq. 4. M^d_θ is the resultant power received with noise. And $SNR^d_\theta = \frac{P^d_\theta}{N}$.

$$M^d_\theta = P^d_\theta + N \tag{3}$$

$$10\log(M^d_\theta) = 10\log(P^d_\theta) + 10\log(1 + \frac{1}{SNR^d_\theta}) \tag{4}$$

The Eq. 5 explodes the view of the random variable $\mathbf{q}_\theta = 10\log(P^d + N) - 10\log(P^d_{ref,\theta})$ (for each sample of specific d) when detected power of unknown target tgt is received at the θ estimator. The value of $P^d_{ref,\theta}$ contains no noise as it is computationally computed this way.

$$\begin{aligned}\mathbf{q}_\theta = 10\log(M^d_{\theta_{tgt}}) - 10\log(P^d_{ref,\theta}) = \\ 10\log(G^d_{t,\theta_{tgt}}) - 10\log(G^d_{t,ref,\theta}) \\ + 10\log(G^d_{r,\theta_{tgt}}) - 10\log(G^d_{r,ref,\theta}) \\ + 10(\log\sigma_{tgt} - \log\sigma_{ref}) \\ - 40(\log R_{tgt} - \log R_{ref}) + 10\log(1 + \frac{1}{SNR^d_\theta})\end{aligned} \tag{5}$$

$$\begin{aligned}var(\mathbf{q}_\theta) = \\ 10var(\log(G^d_{t,\theta_{tgt}})) - 10var(\log(G^d_{t,ref,\theta})) \\ + 10var(\log(G^d_{r,\theta_{tgt}})) - 10var(\log(G^d_{r,ref,\theta})) \\ + 10var(\log(1 + \frac{1}{SNR^d_\theta}))\end{aligned} \tag{6}$$

When $\theta_{tgt} \neq \theta$, the amount of $10\log(G^d_{t,\theta_{tgt}}) - 10\log(G^d_{t,ref,\theta}) + 10\log(G^d_{r,\theta_{tgt}}) - 10\log(G^d_{r,ref,\theta})$ varies when d varies, then $var(\mathbf{q}_\theta)$ is not minimum. Correct value of $var(\mathbf{q}_\theta)$ depends on the antenna radiation patterns as described in Eq. 6.

When the θ estimator is analyzed, where $\theta = \theta_{tgt}$, the random variable $\mathbf{q}_\theta$ is given by Eq. 7.

$$\begin{aligned}\mathbf{q}_\theta = 10\log(M^d_{\theta_{tgt}}) - 10\log(P^d_{ref,\theta}) = \\ 10(\log\sigma_{tgt} - \log\sigma_{ref}) \\ - 40(\log R_{tgt} - \log R_{ref}) + 10\log(1 + \frac{1}{SNR^d_\theta})\end{aligned} \tag{7}$$

The variance of the random variable $\mathbf{q}_\theta$ is given by Eq. 8.

$$var(\mathbf{q}_\theta) = 10var(\log(1 + \frac{1}{SNR^d_\theta})) \tag{8}$$

Comparing Eq. 8 to Eq. 6 shows that $var(\mathbf{q}_\theta)$ of θ estimator when $\theta = \theta_{tgt}$ is smallest among $var(\mathbf{q}_\theta)$ of all estimators.

From the Eq. 8, it is also possible to infer that the higher the SNR, the sample values to the variance are smaller, and then are more stable, that causes $var(\mathbf{q}_\theta)$ to be closer to zero, and azimuth estimation to be more precise as will be seen in the simulation section.

A closed mathematical form for the variance of the azimuth estimation requires a description of the probability density function of each $var(\mathbf{q}_\theta)$ of θ estimator which depends on the antenna radiation patterns. This work is being investigated as future work. The $var(\mathbf{q}_\theta)$ is the ordinate value of a θ estimator; it is not the variance of the azimuth estimation.

It is always possible to find such a best estimator defining the set of estimators to include all possible azimuth output. Of course, the set of estimators is finite and they do no cover all possible real values of azimuth. Although, via simulation, we detected that $var(\mathbf{q}_\theta)$ is still minimum at the θ estimator where $\theta \approx \theta_{tgt}$, being θ_{tgt} the real target azimuth.

IV. The Algorithm

Let the configuration contains just one target for simplicity in the description of algorithm but this processing can be extended to more than one target as the algorithm performs similarly at a specific range bin and azimuth sector.

During the scan of space ahead, the radar points the antenna to all directions d. Each pointing collects (or not) a detection at the target range bin. Let the set of detections be P_i^d.

The algorithm computes the samples of the random variable $\mathbf{q}_\theta$ from Eq. 1 with the set of detections P_i^d, all samples relative to a same θ estimator and a same target. Once samples are computed, the variance is calculated for each estimator. The estimation of the target azimuth is the θ value relative to the θ estimator with the least variance.

To compute the samples by Eq. 5, the values of $P_{ref,\theta}^d$ must be calculated for all antenna direction d and the set of pre-defined azimuths θ. The calculations are done at design time, and stored in a lookup table to optimize the execution of the signal processing stage.

V. Comparison Against the Centroid Algorithm

The centroid algorithm requires that the detected powers of a target to be all from main lobes; it is necessary because the target azimuth estimation is done via an interpolation procedure of the detections powers over a same main lobe curve. This characteristics is only possible when antenna beam patterns for all pointing directions overlap in a condition known as orthogonal mode in which a main lobe for a direction intersect the main lobe of an adjacent direction at 3 dB level.

For the simulation in this paper, the antenna beam patterns were designed in a mode that overlap in a condition more than the orthogonal mode to optimize the requirements for the centroid algorithm to work. A close view of this patterns can be seen in Fig. 2. Such overlap patterns enable a target to be detected by at least three main lobes.

The least estimator algorithm overcomes theoretically the limitation of working only in the overlap condition of the main lobes because detections can be from main and side lobes.

The increased number of detections used makes the variance of each θ estimator more accurate, so the measurement of the target azimuth is more accurate compared to the centroid results.

Also some authors have shown that an analytical mathematical beam shape description improves the target azimuth estimation when the centroid algorithm is used. The current proposed algorithm performs well over beam shaped analytically or simulated.

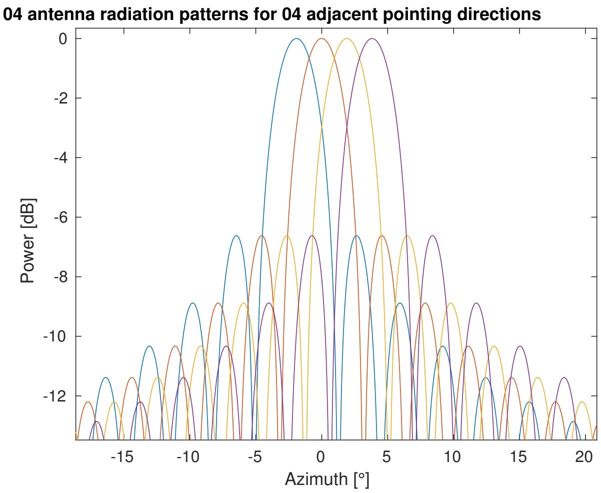

Fig. 2. Close view of 04 adjacent main lobes of 04 antenna pointing directions in which it is possible to see that a target, in any azimuth position, is illuminated by at least three main lobes.

VI. Simulation

A. Description of the simulation

The simulation will show the precision and accuracy of the target azimuth estimation when a unique target is positioned in the azimuths in the range $[0; 47°]$ and with SNR in the range $[10; 30]$ dB.

Target was modeled as a complex number with power given by Eq. 9, and random phase uniformly distributed over $[0; 2\pi]$. Noise power was implemented with 0dB with normal distribution in in-phase and quadrature components.

$$P_{\theta_{tgt}}^d \propto G_{t,\theta_{tgt}}^d G_{r,\theta_{tgt}}^d SNR + N \tag{9}$$

For each pair of azimuth and SNR, the simulation estimates the target azimuth, and calculates its accuracy and precision. The results of accuracy are plot in a 2D coloured map to show evolution over the variation of the parameters. Each algorithm has also two associated figures that are a cut of the respective 2D coloured map:

1) one with target simulated with fixed SNR and different azimuths to show how algorithm behaves varying azimuth; and
2) other with target simulated with fixed azimuth and different SNR to show how algorithm behaves varying SNR.

The two type "1)" figures (from the two algorithms) are overploted for better comparison. The same is done to the type "2)" figures.

Many simulations are done for a same target position and condition. For the target with azimuth θ_{tgt} and SNR, the error E in a measurements of the i-th simulation, where the azimuth calculated is θ, is computed as $E_{tgt}^i = \theta - \theta_{tgt}$. Repeating measurements for the same target condition provides a data set $\{E_{tgt}^i\}$. The accuracy and precision for the target tgt is given by: accuracy=$|mean(\{E_{tgt}\})|$, precision=$var(\{E_{tgt}\})$.

The centroid algorithm was implemented based on [3].

B. Results

From the simulations, the map of accuracy for the current proposed algorithm is shown in Fig. 3, and the map of accuracy for the centroid algorithm is shown in Fig. 4. The map of accuracy is a summary, where each pixel of the map is the resume of accuracy of a target at azimuth and SNR from a batch of simulation carried to generate data to the statistics.

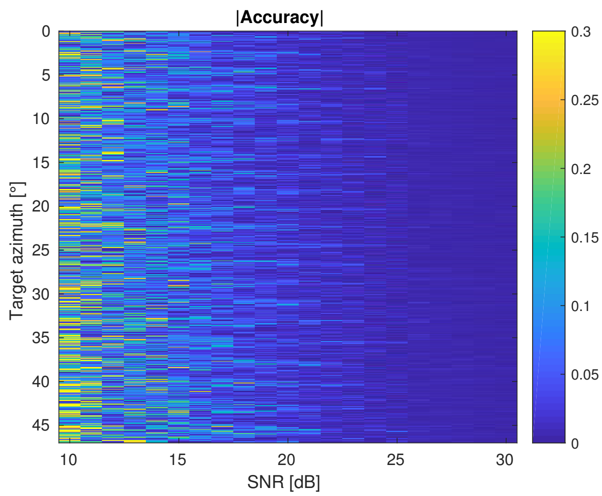

Fig. 3. Map of accuracy of target azimuth varying target SNR and target azimuth, using proposed algorithm.

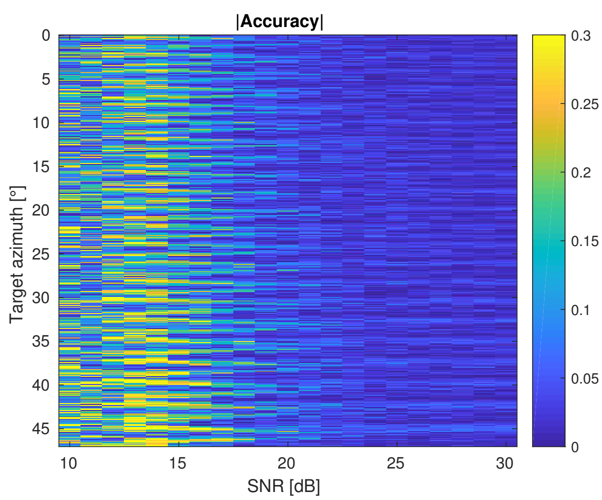

Fig. 4. Map of accuracy of target azimuth varying target SNR and target azimuth, using the centroid algorithm.

Visually it is possible to see that least variance estimator algorithm has better accuracy and precision by having less yellow regions and more darker blue ones to the right. Numerically, the least variance estimator algorithm has 67% better accuracy than the centroid algorithm. Values of accuracy of the least variance estimator algorithm are 0.33 times the values of the centroid algorithm.

For a direct comparison of the two algorithms, Fig. 5 and Fig. 6 show an overplot of the two algorithms. In Fig. 5, the target is at 7.20^0 with SNR in the range $[10; 30]$ dB. In Fig. 6, the target SNR is set to 25dB with azimuth in the range $[0^0; 47^0]$. For each abscissa mark it is shown accuracy, precision and number of detections.

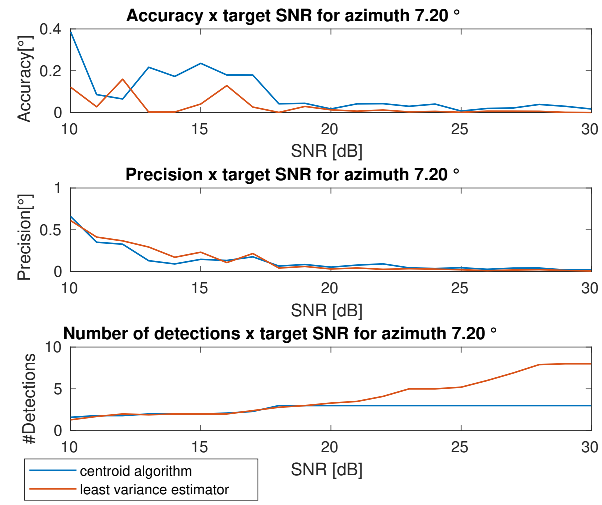

Fig. 5. Accuracy, precision and number of detections for the target in the same azimuth (7.20°), using proposed algorithm.

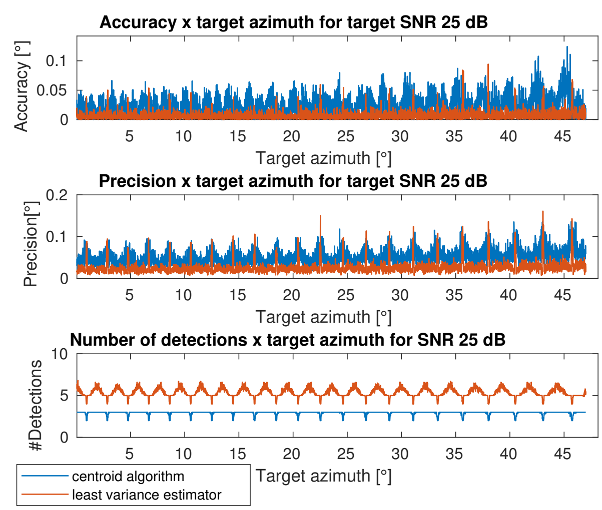

Fig. 6. Accuracy, precision and number of detections for the target in the same SNR ($25dB$), using proposed algorithm.

In Fig. 5 it is seen that the least variance estimator algorithm has better accuracy, and that both algorithms have similar precision but least variance estimator algorithm improves in higher SNR because, as can seen in the #Detections subgraph, the number of detections increases while the centroid algorithm uses always at most three detections.

Another look into the performance comparison in Fig. 6 shows that least variance estimator has better performance overall with better accuracy, better precision and that it uses more detections from a same target.

Fig. 6 also shows that both algorithms lose accuracy and precision when the target is not in the center of a main lobe compared when in the center of a main lobe.

VII. Conclusion

We have proposed a new algorithm for target azimuth estimation that improves over centroid algorithm with better accuracy and precision mainly because it uses more target detections that come from main and side lobes.

To the knowledge of the literature, this is the first algorithm that uses data from side lobes detections.

Also comparing with the execution of the centroid implementation in [3], it is not necessary to do mathematical work to define a mathematical description of the main lobe. The measurements of the antenna gain in annecoic chamber can be directly used. It is to take into consideration that the description of the gains of the antenna are all pre-computed. This characteristics frees the radar designer to have to mathematically describe the antenna main lobe.

Another consideration is that least variance estimator algorithm can be easily implemented in hardware description language for the radar signal processing as it uses a variance as metric for the procedure of azimuth calculation.

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